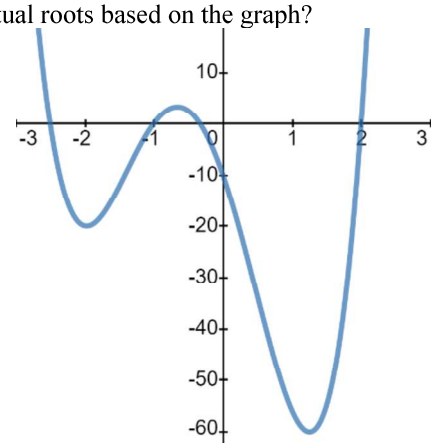
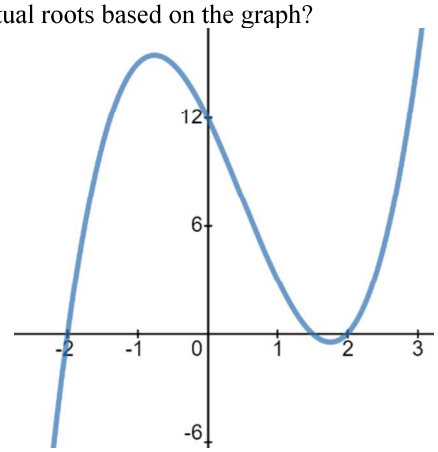
Homework 4.2

$f(x) = 6x^4 + 11x^3 - 24x^2 - 39x - 10$	$g(x) = 2x^3 - 3x^2 - 8x + 12$
1. Make a list of all the possible rational roots of $f(x)$. PRR = $\frac{\pm1, \pm2, \pm5, \pm10}{\pm1, \pm2, \pm3, \pm6}$	4. Make a list of all the possible rational roots of $g(x)$. PRR = $\frac{\pm1, \pm2, \pm3, \pm4, \pm6, \pm12}{\pm1, \pm2}$
2. Which of the possible rational roots from above appear to be actual roots based on the graph?  HPRR = $-5/2, -1, -\frac{1}{3}, 2$	5. Which of the possible rational roots from above appear to be actual roots based on the graph?  HPRR = $-2, \frac{3}{2}, 2$
3. Use synthetic division to find the actual roots of $f(x)$. 2 | 6 11 −24 −39 −10 0 12 46 44 10 6 23 22 5 | 0 −1 | 0 −6 −17 −5 6 17 5 | 0 −5/2 | 0 −15 −5 6 2 | 0 $f(x) = (x-2)(x+1)(x+5/2)(6x+2)$ $6x+2=0$ $6x=-2$ $x=-1/3$ Roots: $-5/2, -1, -\frac{1}{3}, 2$	6. Use synthetic division to find the actual roots of $g(x)$. −2 | 2 −3 −8 12 0 −4 14 −12 2 −7 6 | 0 2 | 0 4 −6 2 −3 | 0 3/2 | 0 3 2 | 0 $g(x) = (x+2)(x-2)(x-\frac{3}{2})\cdot 2$ Roots: $-2, \frac{3}{2}, 2$

A table of values for a polynomial function, $h(x)$, defined by the equation $h(x) = ax^3 + 5x^2 - 12x + c$. The only roots of $h(x)$ lie on the interval $-3 < x < 2$.

x	−3	−2	−1	0	1	2
$h(x)$	−77	0	15	4	3	48

7. Based on the values in the table, can it be determined that $a > 0$ or $a < 0$? Give a reason for your choice.

- $\lim_{x \to -\infty} h(x) = -\infty$
- $\lim_{x \to \infty} h(x) = \infty$
- $h(x)$ has ODD DEGREE of 3

$\therefore a > 0.$

8. What is the value of c in the equation of $h(x)$? Give a reason for your answer.

The y-coordinate of the y-intercept is the same as the constant term.
$h(x)$ has y-intercept of $(0,4)$, thus $c = 4$.

9. Find the value of a using the fact that $h(1) = 3$. Show your work.

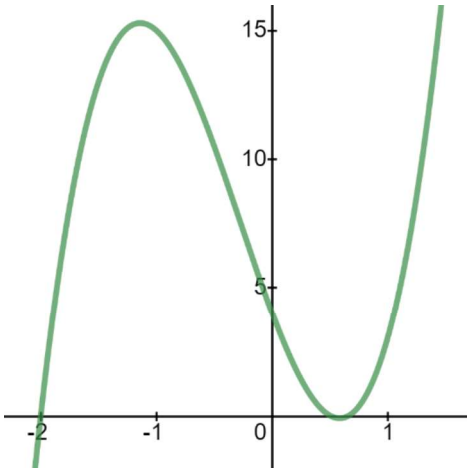
$h(x) = ax^3 + 5x^2 - 12x + 4$
$3 = a(1)^3 + 5(1) - 12(1) + 4$
$3 = a + 5 - 12 + 4$
$3 = a - 3$
$\boxed{6 = a}$

10. Make a list of all the possible rational roots of $h(x)$ now that the values of a and c are known. Then, investigate the graph and identify which three possible rational roots are the most probable rational roots.

$PRR = \dfrac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 2, \pm 3, \pm 6}$

HPRR = $-2, \frac{1}{2}, \frac{2}{3}$

11. Use synthetic division to show that the most probable roots you identified in question 10 are, in fact, rational roots of $h(x)$.



-2	6	5	−12	4
	0	−12	14	−4
$\frac{1}{2}$	6	−7	2	0
	0	3	−2	
$\frac{2}{3}$	6	−4	0	
	0	4		
	6	0		

$h(x) = (x+2)(x-\frac{1}{2})(x-\frac{2}{3}) \cdot 6$

$\boxed{\text{Roots} = -2, \frac{1}{2}, \frac{2}{3}}$