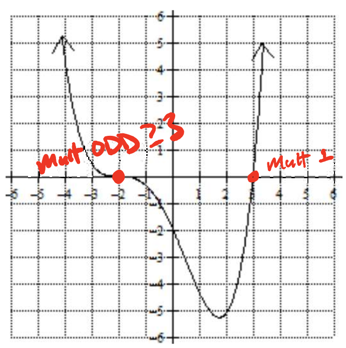
Homework 3.4

For exercises 1 – 3, study the graphs of the two functions that are given. Then, answer the questions that follow.

1. $F(x)$



Identify the zeros of the function and their multiplicities.

$x=-2$, ODD multiplicity ≥ 3

$x=3$, ODD mult $=1$

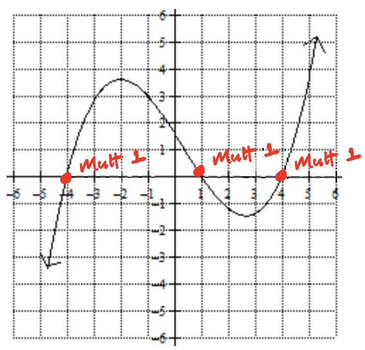
Based on the sum of the multiplicities, what type of function is $F(x)$?

EVEN DEGREE ≥ 4

What are the guaranteed factors of $F(x)$?

$(x+2)$ and $(x-3)$

2. $G(x)$



Identify the zeros of the function and their multiplicities.

$x=-4, 1,$ and 4 each with ODD mult $=1$

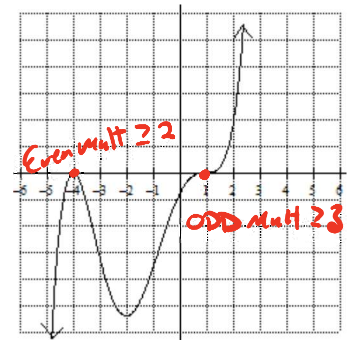
Based on the sum of the multiplicities, what type of function is $G(x)$?

ODD DEGEE $=3$

What are the guaranteed factors of $G(x)$?

$(x+4)$, $(x-1)$ and $(x-4)$

3. $H(x)$



Identify the zeros of the function and their multiplicities.

$x=-4$, EVEN mult ≥ 2

$x=-1$, ODD mult ≥ 3

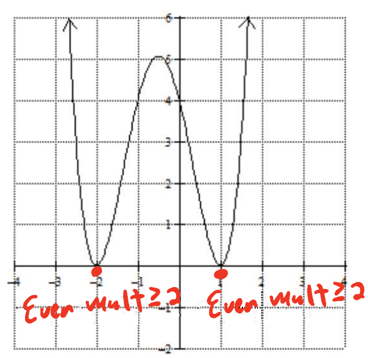
Based on the sum of the multiplicities, what type of function is $H(x)$?

ODD DEGREE ≥ 5

What are the guaranteed factors of $H(x)$?

$(x+4)$ and $(x-1)$

Pictured to the right is the graph of a polynomial function, $g(x)$. Use the graph to answer questions 4 – 8.



4. What type of function is $g(x)$. Completely justify your reasoning.

$x=-2$ and $x=1$ are zeros of $g(x)$ and have Even mult ≥ 2 because $g(x)$ is tangent to the x-axis at $x=-2$ and $x=1$.

The sum of the multiplicities is Even ≥ 4.

$\therefore$ $g(x)$ is even degree ≥ 4.

5. Write an equation in factored form for $g(x)$.

$g(x) = (x+2)^2(x-1)^2$

6. Multiply the equation you wrote in exercise 5 in the form $g(x) = ax^4 + bx^3 + cx^2 + dx + e$.

$g(x) = (x^2+4x+4)(x^2-2x+1)$

$= x^4 + 4x^3 + 4x^2 - 2x^3 - 8x^2 - 8x + x^2 + 4x + 4$

$g(x) = x^4 + 2x^3 - 3x^2 - 4x + 4$

7. Based on the constant term of the equation you produced in exercise 6, why is this a reasonable equation for $g(x)$ graphically?

The constant term in $g(x)$, 4, graphically identifies the y-coordinate of the y-intercept $(0,4)$.

8. Based on the graph of $g(x)$, explain why we know that $x = -2$ is a zero of $g(x)$ whose multiplicity is 2. Use synthetic division on the equation you wrote in exercise 6 to confirm this fact algebraically. Explain why your work confirms this fact.

✓ $\boxed{-2}$	1	2	−3	−4	4
	0	−2	0	6	−4
✓ $\boxed{-2}$	1	0	−3	2	$\boxed{0}$
	0	−2	4	−2	
X $\boxed{-2}$	1	−2	1	$\boxed{0}$	
	0	−2	8		
	1	−4	$\boxed{9}$		

$x=-2$ has an even multiplicity of ≥ 2 because $g(x)$ is tangent to x-axis at $x=-2$.

Algebraically, $x=-2$ has mult $=2$ because $(x+2)$ is a factor of $g(x)$ twice.

9. Using synthetic division, describe what the graph of $h(x) = x^4 - 6x^2 - 8x - 3$ looks like at the zero of $x = -1$. Show your work and explain your reasoning.

✓ $\boxed{-1}$	1	0	−6	−8	−3
	0	−1	1	5	3
✓ $\boxed{-1}$	1	−1	−5	−3	$\boxed{0}$
	0	−1	2	3	
✓ $\boxed{-1}$	1	−2	−3	$\boxed{0}$	
	0	−1	3		
X $\boxed{-1}$	1	−3	$\boxed{0}$		
	0	−1			
	1	$\boxed{-4}$			

$\boxed{\text{x = −1 is a zero 3 times, therefore h(x) cross the x-axis and changes concavity at x = −1}}$

10. For what value of m is the value $x = 2$ a zero of the function $f(x) = -2x^3 + mx^2 + x - 6$?

r=0

$\boxed{2}$	−2	m	1	−6
	0	−4	2m−8	4m−14
	−2	m−4	2m−7	$\boxed{4m-20}$

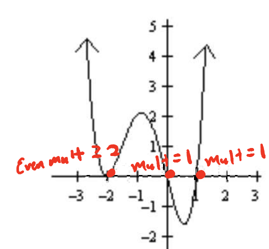
$4m - 20 = 0$
$4m = 20$
$m = 5$

11. The value $x = -2$ is a zero of the function $g(x) = x^4 + 3x^3 - 4x$. Use synthetic division to determine the multiplicity of the root.

✓ $\boxed{-2}$	1	3	0	−4	0
	0	−2	−2	4	0
✓ $\boxed{-2}$	1	1	−2	0	$\boxed{0}$
	0	−2	2	0	
x $\boxed{-2}$	1	−1	0	$\boxed{0}$	
	0	−2	6		
	1	−3	$\boxed{6}$		

$x=-2$ is a root of multiplicity 2.

12. The graph pictured to the right is the graph of $g(x) = x^4 + 3x^3 - 4x$. Explain how your work in exercise 11 is verified by the behavior of the graph at $x = -2$.



The graph of $g(x)$ is tangent to the x-axis at $x=-2$, which implies even multiplicity, ≥ 2. Problem #11 showed $(x+2)$ was a factor twice.