Homework 3.3

1. Suppose the function $f(x) = -2x^3 + 2x - 1$ is divided by the factor $(x + 2)$. Which is greater: the value of $f(-2)$ or the remainder when $f(x)$ is divided by $(x + 2)$? Show your work.

When $x+2=0$, then $x=-2$.

According to the Remainder Theorem $f(-2)$ = the remainder when $\frac{f(x)}{x+2}$

$= -x^3+x^2+2x-1$

2. Suppose the function $g(x) = x^2 - x^3 + 2x - 1$ is divided by the factor $(x - 1)$. Which is greater: the value of $g(-1)$ or the remainder when $g(x)$ is divided by $(x - 1)$? Show your work.

-1	-1	1	2	-1
	0	1	-2	0
	-1	2	0	-1

1	-1	1	2	-1
	0	-1	0	2
	-1	0	2	1

The remainder of $\frac{g(x)}{x-1}$ is greater than $g(-1)$

3. For what value of k will the function $h(x) = -x^3 - 2x^2 + kx - 4$ have a remainder of -2 when divided by the factor $(x - 2)$?

2	-1	-2	k	-4
	0	-2	-8	2k-16
	-1	-4	k-8	2k-20

$2k-20=-2$

$2k=18$

$k=9$

$h(x) \div (x-2)$ will have a remainder of -2 when $k=9$

4. For what value of m will the function $g(x) = mx^2 - 3x + x^3 + 2$ have a remainder of 2 when divided by the factor $(x + 1)$?

$g(x) = x^3 + mx^2 - 3x + 2$

-1	1	m	-3	2
	0	-1	1-m	m+2
	1	m-1	-2-m	m+4

$m+4=2$

$m=-2$

$g(x) \div (x+1)$ will have a remainder of 2 when $m=-2$

5. For what value of m will the function $p(x) = 2x^4 + mx - 4$ have a remainder of 0 when divided by the factor $(x + 2)$?

-2	2	0	0	m	-4
	0	-4	8	-16	-2m+32
	2	-4	8	m-16	-2m+28

$-2m+28=0$

$28=2m$

$14=m$

$p(x) \div (x+2)$ will have remainder of 0 when $m=14$

For problems 6 – 9, use synthetic division to determine if the given factor is a factor of the function $f(x) = x^3 + 3x^2 - 10x - 24$. Show your work and write yes or no.

6. $(x+4)$

-4	1	3	-10	-24
	0	-4	4	24
	1	-1	-6	0

YES $(x+4)$ is a factor of $f(x)$

7. $(x-3)$

3	1	3	-10	-24
	0	3	18	24
	1	6	8	0

YES $(x-3)$ is a factor of $f(x)$

8. $(x+6)$

-6	1	3	-10	-24
	0	-6	18	-48
	1	-3	8	-72

NO $(x+6)$ is NOT a factor of $f(x)$

9. $(x+2)$

-2	1	3	-10	-24
	0	-2	-2	24
	1	1	-12	0

YES $(x+2)$ is a factor of $f(x)$

10. What reasoning did you use in exercises 6 – 9 to determine if the factor what a factor of $f(x)$ or not?

The Factor Theorem If the remainder = 0, then the divisor is a factor of $f(x)$.

11. $(x+2)$ ($x=-2$) and $(x-3)$ ($x=3$) are the only factors of the function $g(x) = x^3 + x^2 - 8x - 12$. How many times is each a factor of $g(x)$? Show your work.

-2	1	1	-8	-12
	0	-2	2	12
	1	-1	-6	0

$g(x) = (x+2)(x^2 - x - 6)$

$g(x) = (x+2)(x-3)(x+2)$

$(x+2)$ is a factor twice

$(x-3)$ is a factor once.

12. If $x = 3$ is a zero of the function $h(x) = -2x^4 - 63x^2 + 19x^3 + 81x - 27$, what factor is a factor of $h(x)$ and how many times is it a factor? Show your work. $h(x) = -2x^4 + 19x^3 - 63x^2 + 81x - 27$

✓ 3

	-2	19	-63	81	-27
	0	-6	39	-72	27

✓ 3

	-2	13	-24	9	0
	0	-6	21	-9	

✓ 3

	-2	7	-3	0
	0	-6	3	

✗ 3

	-2	1	0
	0	-6	
	-2	-5	

$(x-3)$ is a factor of $h(x)$ three times

13. If $(x-2)$ is a factor of $f(x) = 2x^3 - 3x^2 - 3x + 2$, what are the other two factors? Show your work using synthetic division.

2	2	-3	-3	2
	0	4	2	-2
	2	1	-1	0

$f(x) = (x-2)(2x^2 + x - 1)$

$= (x-2)[2x^2 + 2x - x - 1]$

$= (x-2)[2x(x+1) - 1(x+1)]$

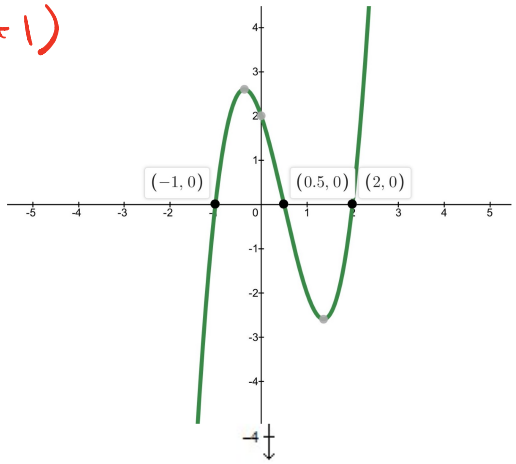
$f(x) = (x-2)(x+1)(2x-1)$

$(2x-1)$ and $(x+1)$ are the other two factors of $f(x)$.

14. Using a graphing calculator, draw the graph of $f(x)$ on the axis provided below. Then, explain why your work in exercise 13 is validated by this graph.

If the factors $(x-2)$, $(2x-1)$ and $(x+1)$ are $\neq 0$ and solved then $x = 2, \frac{1}{2}$, and -1.

The graph also shows the zeros are $x = 2, \frac{1}{2}$, and -1.



15. For what value of k would the factor $(x+3)$ be a factor of $f(x) = -3x^3 + kx^2 + 20x - 12$? Show your work.

R = 0

-3

-3	K	20	-12
0	9	$-3K-27$	$9K+21$
-3	$K+9$	$-3K-7$	$9K+9$

$9K+9=0$

$9K=-9$

$K=-1$

16. For what value of k would the factor $(x-1)$ be a factor of $g(x) = 3x^4 + 2x^3 + kx^2 - 2x + 1$? Show your work.

R = 0

1

3	2	K	-2	1
0	3	5	$K+5$	$K+3$
3	5	$K+5$	$K+3$	$K+4$

$K+4=0$

$K=-4$