Homework 2.3

1. Complete the table below so that $G(x)$ is a function, but is NOT a one-to-one function. Specifically explain why your example is not a one-to-one function.

x	–2	–1	$-1/2$	0	4
$G(x)$	10	0	–1	–1	–7

The y-value of –1 is paired with two x-values, $-1/2$ and 0
∴ G(x) is not one-to-one

2. Given that $f(x)$ and $g(x)$ are two one-to-one functions and that $f(2) = 3$ and $g(3) = 1$. Are f and g inverses of each other or not? Explain why or why not?

f and g are not inverses of each other. If f(2)=3, then $f^{-1}(3)=2$, so (3,2) would have to be a point on g, but g(3)=1

3. Analytically determine if $f(x) = 2x + 1$ and $g(x) = 0.5x – 1$ are inverses of each other or not. Show your work and explain your reasoning based on that work.

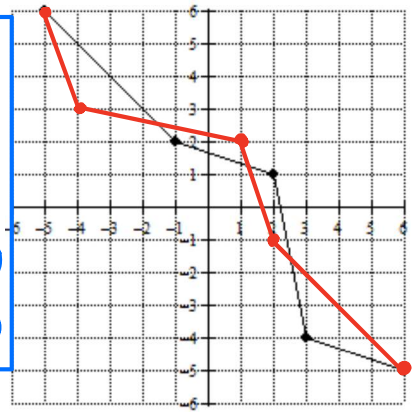
$f(g(x)) = f(\frac{1}{2}x-1)$
$= 2(\frac{1}{2}x-1)+1$
$= x-2+1$
$= x-1$

$f(g(x)) \neq x$
∴ f and g are not inverses

4. Graphed below are two functions, F and G. Determine if F^{-1} and/or G^{-1} exist or not. Explain why why or why not. If the inverse does exist, then also draw a graph of the inverse on the grid.

F(x)	$F^{-1}(x)$
(–5,6)	(6,–5)
(–1,2)	(2,–1)
(2,1)	(1,2)
(3,–4)	(–4,3)
(6,–5)	(–5,6)

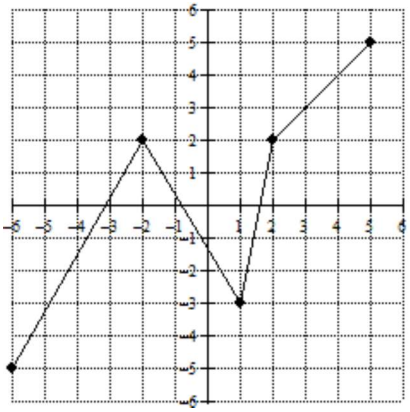
F(x)



The graph of F(x) passes the HLT (or b/c it is always decreasing on Domain)

∴ F(x) is 1-1.
∴ $F^{-1}(x)$ exists

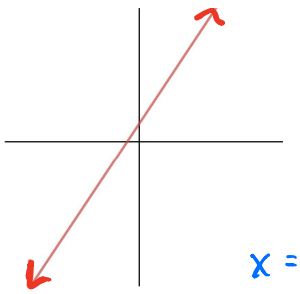
G(x)



The graph of G(x) does not pass the HLT
∴ G(x) is not 1-1
∴ $G^{-1}(x)$ does not exists

For problems 5 – 8, use a graphing calculator to draw a rough sketch of the graph of the function given. Draw a sketch of the graph on a set of axes. Then, (a) determine if the function is a one-to-one function or not, explaining your reasoning for each one and (b) if the function is one-to-one, find the equation of the inverse of the function. Show all of your work and use complete sentences in your statements of reasoning.

5. $f(x) = \dfrac{3x+1}{2}$



The graph of f(x) passes the HLT.

∴ f(x) is 1-1

∴ $f^{-1}(x)$ exists

$x = \dfrac{3y+1}{2}$

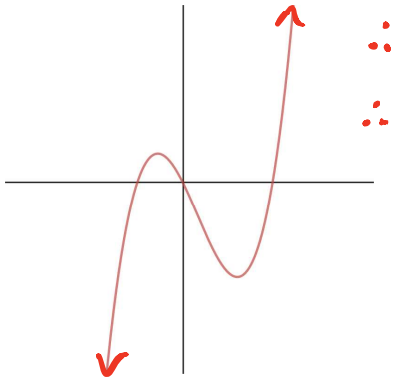
$2x = 3y+1$

$2x-1 = 3y$

$\dfrac{2x-1}{3} = y$

$f^{-1}(x) = \dfrac{2x-1}{3}$

6. $f(x) = x^3 - x^2 - 2x$

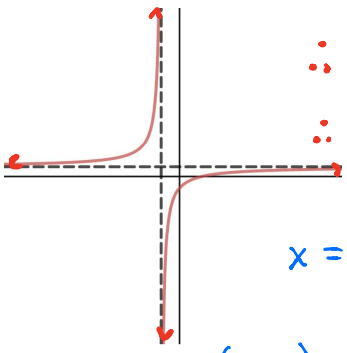


The graph of f(x) fails the HLT

∴ f(x) is not 1-1

∴ $f^{-1}(x)$ does not exist.

7. $f(x) = \dfrac{x-3}{x+2}$



The graph of f(x) passes the HLT.

∴ f(x) is 1-1

∴ $f^{-1}(x)$ exists

$x = \dfrac{y-3}{y+2}$

$x(y+2) = y-3$

$xy+2x = y-3$

$2x+3 = y-xy$

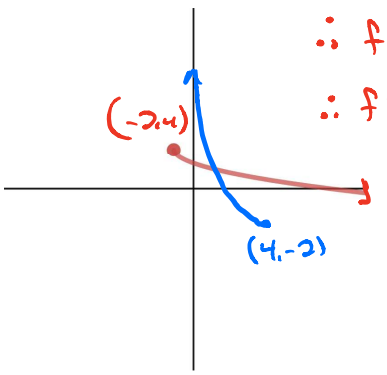
$2x+3 = y(1-x)$

$\dfrac{2x+3}{1-x} = y$

$f^{-1}(x) = -\dfrac{2x+3}{x-1}$

(or Equivalent rational)

8. $f(x) = -\sqrt{x+2}+4$



The graph of f(x) passes the HLT.

∴ f(x) is 1-1

∴ $f^{-1}(x)$ exists

$x = -\sqrt{y+2}+4$

$x-4 = -\sqrt{y+2}$

$(-(x-4))^2 = (\sqrt{y+2})^2$

$(x-4)^2 = y+2$

$(x-4)^2 - 2 = y$

$f^{-1}(x) = (x-4)^2 - 2, \quad x \le 4$