

Name _____

Date _____

Period _____

Homework 10.2 Part 1

Analytically show that the equations below represent trigonometric identity statements.

1. $\sec^2 \theta (1 - \cos^2 \theta) = \tan^2 \theta$

$$\sec^2 \theta (\sin^2 \theta) =$$

$$\frac{1}{\cos^2 \theta} \cdot \sin^2 \theta =$$

$$\frac{\sin^2 \theta}{\cos^2 \theta} =$$

$$\tan^2 \theta = \tan^2 \theta$$

2. $\cos x (\sec x - \cos x) = \sin^2 x$

$$\cos x \cdot \sec x - \cos x \cdot \cos x =$$

$$1 - \cos^2 x =$$

$$\sin^2 x = \sin^2 x$$

3. $\cos \theta + \sin \theta \tan \theta = \sec \theta$

$$\frac{\cos \theta}{\cos \theta} + \sin \theta \cdot \frac{\sin \theta}{\cos \theta}$$

$$\frac{\cos^2 \theta + \sin^2 \theta}{\cos \theta} =$$

$$\frac{1}{\cos \theta} =$$

$$\sec \theta = \sec \theta$$

4. $(1 - \cos \alpha)(\csc \alpha + \cot \alpha) = \cos \alpha \tan \alpha$

$$\csc \alpha + \cot \alpha - \cos \alpha \csc \alpha - \cos \alpha \cot \alpha =$$

$$\frac{1}{\sin \alpha} + \frac{\cos \alpha}{\sin \alpha} - \cos \alpha \cdot \frac{1}{\sin \alpha} - \cos \alpha \cdot \frac{\cos \alpha}{\sin \alpha} =$$

$$\frac{1 - \cos^2 \alpha}{\sin \alpha} = \cos \alpha \cdot \frac{\sin \alpha}{\cos \alpha}$$

$$\frac{\sin^2 \alpha}{\sin \alpha} =$$

$$\sin \alpha = \sin \alpha$$

5. $\sin x (\csc x + \sin x \sec^2 x) = \sec^2 x$

$$\begin{aligned} \sin x \cdot \csc x + \sin^2 x \sec^2 x &= \sec^2 x \\ 1 + \sin^2 x \cdot \frac{1}{\cos^2 x} &= \sec^2 x \\ 1 + \frac{\sin^2 x}{\cos^2 x} &= \\ 1 + \tan^2 x &= \\ \sec^2 x &= \sec^2 x \end{aligned}$$

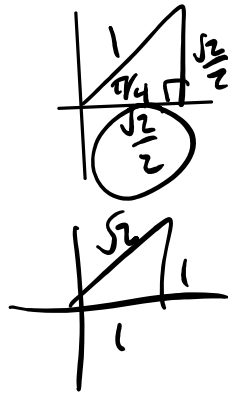
$$\begin{aligned} \frac{(1+\sec\theta)\tan\theta}{(1+\sec\theta)} + \frac{\tan\theta}{1+\sec\theta} &= 2 \csc\theta \\ \frac{(1+\sec\theta)\tan\theta + \tan\theta}{(1+\sec\theta)\tan\theta} &= \\ \frac{1 + 2\sec\theta + \sec^2\theta + \tan^2\theta}{(1+\sec\theta)\tan\theta} &= \\ \frac{2\sec\theta + \sec^2\theta + \sec^2\theta}{(1+\sec\theta)\tan\theta} &= \\ \frac{2\sec\theta + 2\sec^2\theta}{(1+\sec\theta)\tan\theta} &= \\ \frac{2\sec\theta(1+\sec\theta)}{(1+\sec\theta)\tan\theta} &= \end{aligned}$$

$$\begin{aligned} \frac{2\sec\theta}{\tan\theta} &= \\ 2\sec\theta \cdot \cot\theta &= \\ 2 \cdot \frac{1}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta} &= \\ 2 \cdot \frac{1}{\sin\theta} &= \\ 2\csc\theta &= 2\csc\theta \end{aligned}$$

If $\theta = \frac{\pi}{4}$, numerically show that the following equations are identities by finding the exact values of each expression. Show your work and leave your answers with rationalized denominators.

6. $\sec^2 \theta + \csc^2 \theta = \sec^2 \theta \csc^2 \theta$

$$\begin{aligned} \frac{\sin^2\theta}{\sin^2\theta \cos^2\theta} + \frac{1}{\sin^2\theta} \frac{\cos^2\theta}{\cos^2\theta} &= \\ \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta \cos^2\theta} &= \\ \frac{1}{\sin^2\theta \cos^2\theta} &= \end{aligned}$$



$$\begin{aligned} \csc^2\theta \sec^2\theta &= \\ \sec^2\theta \csc^2\theta &= \sec^2\theta \csc^2\theta \end{aligned}$$

8. $\cos \theta + \sin \theta \tan \theta = \sec \theta$

$$\begin{aligned} \cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4}\right) \tan\left(\frac{\pi}{4}\right) &= \sec\left(\frac{\pi}{4}\right) \\ \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot 1 &= \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \\ \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} &= \frac{2\sqrt{2}}{2} \\ \sqrt{2} &= \sqrt{2} \end{aligned}$$

9. $\frac{\sin^2 \theta}{\cos \theta} = \sec \theta - \cos \theta$

$$\begin{aligned} \frac{\sin^2\left(\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}\right)} &= \sec\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{4}\right) \\ \frac{\left(\frac{\sqrt{2}}{2}\right)^2}{\frac{\sqrt{2}}{2}} &= \sqrt{2} - \frac{\sqrt{2}}{2} \\ \frac{\frac{2}{4}}{\frac{\sqrt{2}}{2}} &= \frac{2\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \\ \frac{2 \cdot \sqrt{2}}{2 \cdot \sqrt{2} \cdot 2} &= \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} &= \frac{\sqrt{2}}{2} \end{aligned}$$