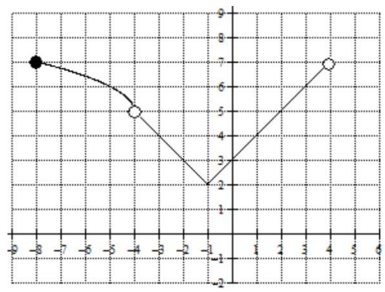
Homework 1.7 Part II

1. The graph of $F(x)$ consists of a piece of a square root function and a piece of an absolute value function.



a. Write the equation of $F(x)$ below.

$$F(x)=\begin{cases}\sqrt{-(x+4)}+5, & -8\le x<-4\\ |x+1|+2, & -4<x<4\end{cases}$$

b. Domain of $F(x)$: $[-8,-4)\cup(-4,4)$

c. Range of $F(x)$: $[2,7]$

d. Use the graph to determine if $F(x)$ is continuous at $x = -4$? If it is not, classify the discontinuity and justify your answer.

I. $F(-4)$ is undefined

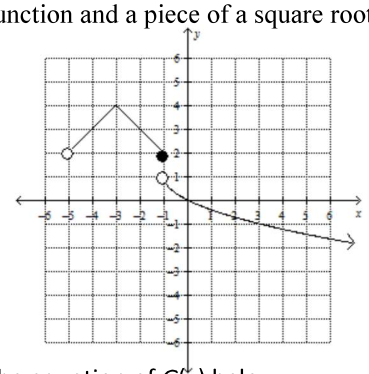
II. $\lim\limits_{x\to -4^-} F(x) = \lim\limits_{x\to -4^+} F(x) = 5$

$\therefore \lim\limits_{x\to -4} F(x)$ exists

III. $\lim\limits_{x\to -4} F(x) \neq F(-4)$

$\therefore F(x)$ has point discontinuity at $x=-4$

2. The graph of $G(x)$ consists of a piece of an absolute value function and a piece of a square root function.



a. Write the equation of $G(x)$ below.

$$G(x)=\begin{cases}-|x+3|+4, & -5<x\le -1\\ -\sqrt{x+1}+1, & x>-1\end{cases}$$

b. Domain of $G(x)$: $(-5,\infty)$

c. Range of $G(x)$: $(-\infty,1)\cup[2,4]$

d. Use your graph to determine if $G(x)$ is continuous at $x = -1$? If it is not, classify the discontinuity and justify your answer.

I. $G(-1)=2$

$\therefore G(-1)$ is defined

II. $\lim\limits_{x\to -1^-} G(x) = 2$

$\lim\limits_{x\to -1^+} G(x) = 1$

$\lim\limits_{x\to -1^-} G(x) \neq \lim\limits_{x\to -1^+} G(x)$

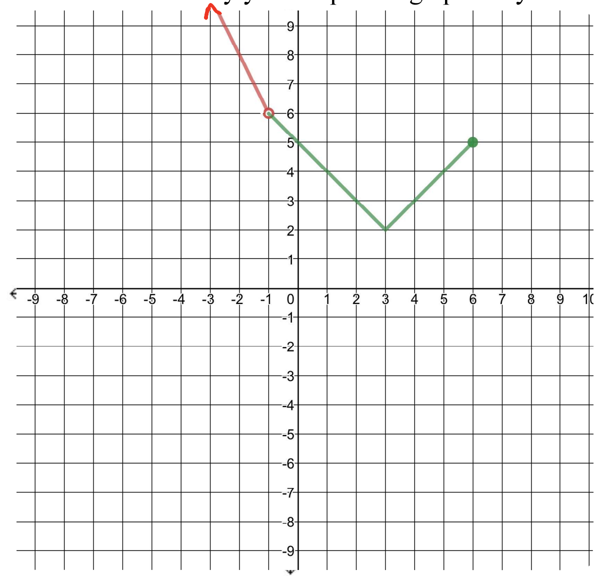
$\therefore \lim\limits_{x\to -1} G(x)$ does not exist

III. $\lim\limits_{x\to -1} G(x) \neq G(-1)$

$\therefore G(x)$ has jump discontinuity at $x=-1$

For each of the piece-wise defined functions below, use the equation to identify and classify any and all discontinuities that may exist in the graph. Justify your responses analytically and then graph the function to verify your responses graphically.

3. $G(x) = \begin{cases} -2x+4, & x < -1 \\ |x-3|+2, & -1 < x \le 6 \end{cases}$



I. $G(-1)$ is undefined

II. $\lim_{x \to -1^-} G(x) = -2(-1)+4 = 2+4 = 6$

$\lim_{x \to -1^+} G(x) = |(-1)-3|+2 = |-4|+2 = 4+2 = 6$

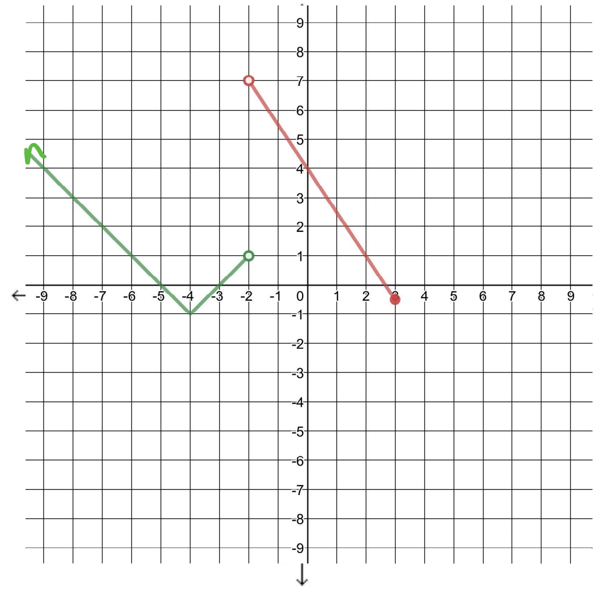
$\therefore \lim_{x \to -1^-} G(x) = \lim_{x \to -1^+} G(x) = 6$

$\therefore \lim_{x \to -1} G(x)$ exists

III $\lim_{x \to -1} G(x) \neq G(-1)$

$\therefore$ G(x) has point Discontinuity at $x = -1$

4. $H(x) = \begin{cases} |x+4|-1, & x < -2 \\ -\frac{3}{2}x + 4, & -2 < x \le 3 \end{cases}$



I. H(-2) is undefined

II. $\lim_{x \to -2^-} H(x) = |(-2)+4| - 1 = |2| - 1 = 2 - 1 = 1$

$\lim_{x \to -2^+} H(x) = -\frac{3}{2}(-2) + 4 = 3 + 4 = 7$

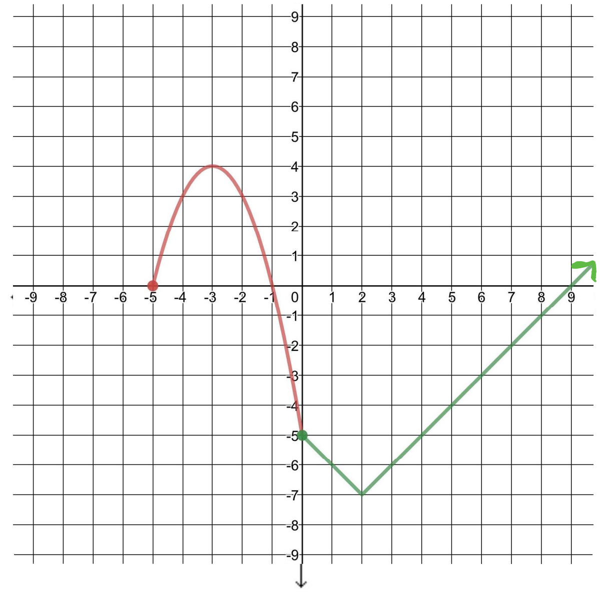
$\therefore \lim_{x \to -2^-} H(x) \neq \lim_{x \to -2^+} H(x)$

$\therefore \lim_{x \to -2} H(x)$ does not exist.

III. $\lim_{x \to -2} H(x) \neq H(-2)$

$\therefore$ H(x) has jump discontinuity at $x = -2$

5. $H(x) = \begin{cases} -(x+3)^2 + 4, & -5 \leq x < 0 \\ |x-2| - 7, & x \geq 0 \end{cases}$



I. $H(0) = |0-2| - 7 = |-2| - 7 = 2 - 7 = -5$

$\therefore$ $H(0)$ is defined

II. $\lim_{x \to 0^-} H(x) = -[(0)+3]^2 + 4 = -[3]^2 + 4 = -9 + 4 = -5$

$\lim_{x \to 0^+} H(x) = |(0)-2| - 7 = |-2| - 7 = 2 - 7 = -5$

$\therefore \lim_{x \to 0^-} H(x) = \lim_{x \to 0^+} H(x) = -5$

$\therefore \lim_{x \to 0} H(x)$ exists

III. $\lim_{x \to 0} H(x) = H(0) = -5$

$\therefore$ $H(x)$ is continuous at $x=0$