Free Response Practice #7
Calculator NOT Permitted

$\sqrt{-(x+3)}+2$

Consider the piecewise functions $f(x)$ and $g(x)$ shown below to answer the following questions.

$$f(x)=\begin{cases}\sqrt{-x-3}+2, & -7<x\le -3\\ -3, & -3<x<3\\ -(x-5)^2+1, & x>3\end{cases}\qquad g(x)=\begin{cases}ax+3, & x<-3\\ x^2-3x, & -3<x<2\\ bx-5, & x>2\end{cases}$$

a. Analytically determine if the function $f(x)$ is discontinuous at $x = -3$ or not. If there is a discontinuity, classify it. Show the work and provide justification that each support your answer.

I. $f(-3)=\sqrt{-(-3+3)}+2=\sqrt{-0}+2=2$

$\therefore f(-3)$ is defined +1

II. $\lim\limits_{x\to -3^-} f(x)=\sqrt{-(-3+3)}+2=\sqrt{-0}+2=2$

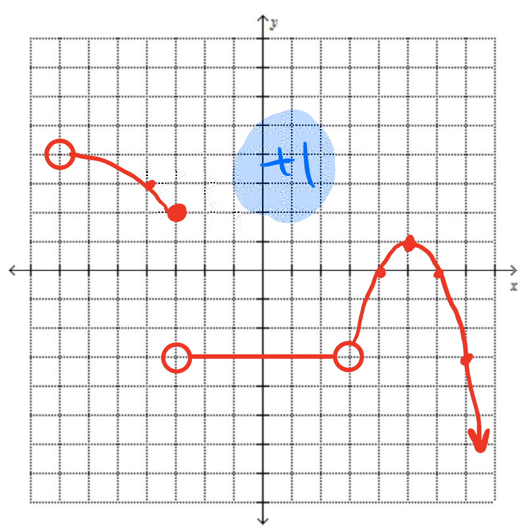
$\lim\limits_{x\to -3^+} f(x)=-3$ +1

$\therefore \lim\limits_{x\to -3} f(x)$ does not exist

III. $\lim\limits_{x\to -3} f(x)\ne f(-3)$ +1

+1 $\therefore f(x)$ has jump discontinuity at $x=-3$

b. Sketch an accurate graph of the piecewise function. Then, based on the graph, identify the domain, range, and determine, providing graphical justification, what type of discontinuity occurs at $x = 3$.



Domain of $f(x)$: $(-7,3)\cup(3,\infty)$ +1/2

Range of $f(x)$: $(-\infty,1]\cup[2,4)$ +1/2

Type of Discontinuity at $x = 3$ with justification:

+1

I. $f(3)$ is undefined

II. $\lim\limits_{x\to 3} f(x)=-3\ \therefore \lim\limits_{x\to 3} f(x)$ exists

III. $f(3)\ne \lim\limits_{x\to 3} f(x)$

$\therefore f(x)$ has point discontinuity at $x=3$

Consider the piecewise functions $f(x)$ and $g(x)$ shown below to answer the following questions.

$$f(x)=\begin{cases}\sqrt{-x-3}+2, & -7<x\le -3\\ -3, & -3<x<3\\ -(x-5)^2+1, & x>3\end{cases} \qquad g(x)=\begin{cases}ax+3, & x<-3\\ x^2-3x, & -3<x<2\\ bx-5, & x>2\end{cases}$$

c. For what values of *a* and *b* would the function $g(x)$ have point discontinuities at $x = -3$ and $x = 2$. Show your work.

I. $g(-3)$ is undefined

II. $\lim_{x\to -3} g(x)$ must exist

$$\lim_{x\to -3^-} g(x)=\lim_{x\to -3^+} g(x)$$

$$a(-3)+3=(-3)^2-3(-3)$$

$$-3a+3=9+9$$

$$-3a+3=18$$

$$-3a=15$$

$$a=-5$$

+1

III $\lim_{x\to -3} g(x)\neq g(-3)$

I. $g(2)$ is undefined

II $\lim_{x\to 2} g(x)$ must exist

$$\lim_{x\to 2^-} g(x)=\lim_{x\to 2^+} g(x)$$

$$(2)^2-3(2)=b(2)-5$$

$$4-6=2b-5$$

$$-2=2b-5$$

$$3=2b$$

$$3/2=b$$

+1

III $\lim_{x\to 2} g(x)\neq g(2)$