

Free Response Practice #6
Calculator NOT Permitted

Consider the equation of the piece-wise defined function, $f(x)$, and the equation of the rational function, $g(x)$, below to answer the questions that follow.

$$f(x) = \begin{cases} -\frac{1}{2}x - 1, & x < -4 \\ 3, & x = -4 \\ -\sqrt{x+4} + 1, & -4 < x \leq 5 \end{cases}$$

open endpoint $(-4, 1)$ $m = -\frac{1}{2}$

$$g(x) = \frac{x-4}{2x+1}$$

$\checkmark(-4, 1)$

a. Analytically determine if the graph of $f(x)$ will have a discontinuity at $x = -4$. If it does, then classify the discontinuity. Justify your reasoning and show your work.

I. $f(-4) = 3 \therefore f(x)$ is defined +1

II. $\lim_{x \rightarrow -4^-} f(x) = -\frac{1}{2}(-4) - 1 = 2 - 1 = 1$

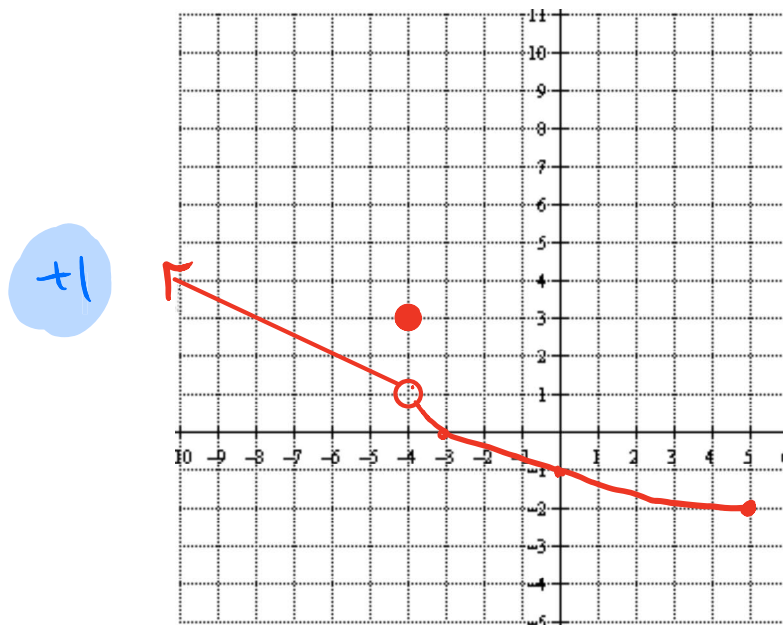
$\lim_{x \rightarrow -4^+} f(x) = -\sqrt{(-4)+4} + 1 = -\sqrt{0} + 1 = 1$ +1

$\therefore \lim_{x \rightarrow -4} f(x)$ exists

III. $\lim_{x \rightarrow -4} f(x) \neq f(-4)$ +1

$\therefore f(x)$ has point discontinuity at $x = -4$ +1

b. Graph $f(x)$. You should be able to verify your analytical conclusion from part a) in the graph.



Consider the equation of the piece-wise defined function, $f(x)$, and the equation of the rational function, $g(x)$, below to answer the questions that follow.

$$f(x) = \begin{cases} -\frac{1}{2}x - 1, & x < -4 \\ 3, & x = -4 \\ -\sqrt{x+4} + 1, & -4 < x \leq 5 \end{cases} \quad g(x) = \frac{x-4}{2x+1}$$

- c. If $f(x) < 0$, then graphically what must be true about $f(x)$. Explain your reasoning. Also, if the graph of $f(x)$ is ever such that $f(x) < 0$, then state on which interval(s) along the x -axis it is so.

• If $f(x) < 0$, then the graph of $f(x)$ is below the x -axis because y is negative below the x -axis. +1

• $f(x) < 0$ on $(-3, 5]$ +1

- d. If $p(x) = -\frac{1}{2}|x+1|$, then for what value(s) of x is $p(x) = f(g(-1))$? Show your work.

$g(-1) = \frac{(-1)-4}{2(-1)+1}$ $= \frac{-5}{-2+1}$ $= \frac{-5}{-1}$ $g(-1) = 5$	$f(g(-1)) = f(5)$ $f(g(-1)) = -2$ <p>(by graph)</p>	$p(x) = f(g(-1))$ $-\frac{1}{2} x+1 = -2$ $ x+1 = 4$ $x+1 = \pm 4$ $x = -1 \pm 4$ $x = -5 \text{ and } x = 3$
+1		+1