Free Response Practice #5
Calculator NOT Permitted

$$f(x)=\begin{cases} x^2+\frac{2}{3}x, & -9<x\le -3 \\ -2x+1, & -3<x<2 \\ x+3, & x>2 \end{cases} \qquad g(x)=\begin{cases} ax+3, & x<-2 \\ x^2+2x, & x\ge -2 \end{cases}$$

Consider the two piece-wise defined functions, $f(x)$ and $g(x)$, above to answer the following questions.

a. Find $f(-9)$, $f(-3)$, $(f+g)(3)$ and the domain of $f(x)$. Show or explain your work for each.

+1/2 $f(-9)$ is undefined b/c $x=-9$ is not in the domain of $f(x)$

+1/2 Domain of $f(x)$ is $(-9,2)\cup(2,\infty)$

+1/2
$$f(-3)=(-3)^2+\tfrac{2}{3}(-3)$$
$$=9-2$$
$$f(-3)=7$$

$$(f+g)(3)=f(3)+g(3)$$
$$=[(3)+3]+[(3)^2+2(3)]$$
$$=6+9+6$$
$$(f+g)(3)=21$$
+1/2

b. Does $f(x)$ have a discontinuity at $x=-3$? If so, classify it. Justify your reasoning.

I. $f(-3)=7$ +1

$\therefore$ $f(-3)$ is defined

II. $\lim_{x\to -3^-} f(x)=(-3)^2+\tfrac{2}{3}(-3)=9-2=7$

$\lim_{x\to -3^+} f(x)=-2(-3)+1=6+1=7$ +1

$\therefore \lim_{x\to -3} f(x)$ exists

III. $\lim_{x\to -3} f(x)=f(-3)=7$ +1

$\therefore$ $f(x)$ is continuous at $x=-3$ +1

$$f(x)=\begin{cases} x^2+\frac{2}{3}x, & -9<x\le -3 \\ -2x+1, & -3<x<2 \\ x+3, & x>2 \end{cases}$$

$$g(x)=\begin{cases} ax+3, & x<-2 \\ x^2+2x, & x\ge -2 \end{cases}$$

Consider the two piece-wise defined functions, *f*(*x*) and *g*(*x*), above to answer the following questions.

c. For what value(s) of a is the graph of $g(x)$ continuous at $x = -2$?

I $g(-2) = (-2)^2 + 2(-2) = 4 - 4 = 0$

$\therefore g(-2)$ is defined.

II $\lim_{x\to -2} g(x)$ must exist

$\lim_{x\to -2^-} g(x) = \lim_{x\to -2^+} g(x)$

+1 $a(-2)+3 = (-2)^2 + 2(-2)$ +1

$-2a+3 = 4-4$

$-2a+3=0$

$-2a=-3$

$a = 3/2$ +1

III $\lim_{x\to -2} g(x) = g(-2)$