

Free Response Practice #25
Calculator Permitted

The table below shows function values for a rational function, $G(x)$. The equation of $G(x)$ is such that $(x + 2)$ and $(x - 1)$ are the only factors in the denominator of the function.

| | | | | | | | | | |
|--------|-------|--------|-----------|--------|----|-------|-----------|-------|-------|
| x | -1000 | -2.001 | -2 | -1.999 | 0 | 0.999 | 1 | 1.001 | 1000 |
| $G(x)$ | 0.998 | 0.333 | Undefined | 0.333 | -1 | -1999 | Undefined | 2001 | 1.002 |

a. Does either factor in the denominator also exist in the numerator? If so, which factor? Give a reason for your answer.

$\lim_{x \rightarrow -2^-} G(x) = \frac{1}{3}$
 $\lim_{x \rightarrow -2^+} G(x) = \frac{1}{3}$
 $G(-2) \neq \frac{1}{3}$

$\therefore G(x)$ has point discontinuity at $x = -2$
 $\therefore (x+2)$ is a factor in both the numerator and denominator.

b. Does either factor of the denominator not exist in the numerator? If so, which factor? Give a reason for your answer.

$\lim_{x \rightarrow 1^-} G(x) = -\infty$
 $\lim_{x \rightarrow 1^+} G(x) = \infty$

$\therefore G(x)$ has a vertical asymptote at $x = 1$
 $\therefore (x-1)$ is a factor of the denominator only.

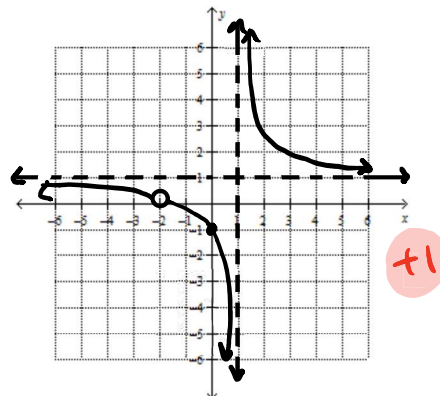
c. Based on the end behavior, where does $G(x)$ have a horizontal asymptote? Give a reason for your answer.

$\lim_{x \rightarrow -\infty} G(x) = 1$
 $\lim_{x \rightarrow \infty} G(x) = 1$

$\therefore G(x)$ has a horizontal asymptote at $y = 1$

d. Sketch a possible graph of the function $G(x)$. Then, state the domain and range of $G(x)$.

Domain: $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$
 Range: $(-\infty, \frac{1}{3}) \cup (\frac{1}{3}, 1) \cup (1, \infty)$



Free Response Practice #25 Grading Rubric

Free Response Part A – 2 points total

_____ 1 The factor $(x + 2)$ also exists in the numerator.

_____ 1 According to the table, values very close to $x = -2$ approach the same y -value, 0.333 or $\frac{1}{3}$.
This means that the graph has a hole in it at the point $(-2, \frac{1}{3})$.

Free Response Part B – 2 points total

_____ 1 The factor $(x - 1)$ does not exist in the numerator.

_____ 1 According to the table, values very close to $x = 1$ to the left approach $-\infty$ and values very close to $x = 1$ to the right approach ∞ . This means that $x = 1$ is a vertical asymptote.

Free Response Part C – 2 points total

_____ 1 For very low values of x , such as $x = -1000$ and for very large value of x , such as $x = 1000$, the y -values of $G(x)$ approach the value of 1 .

_____ 1 Thus, the horizontal asymptote is $y = 1$.

Free Response Part D – 3 points total

_____ 1 Sketch of graph shown to the right

_____ 1 Domain: $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$

_____ 1 Range: $(-\infty, \frac{1}{3}) \cup (\frac{1}{3}, 1) \cup (1, \infty)$

