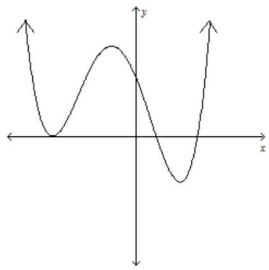
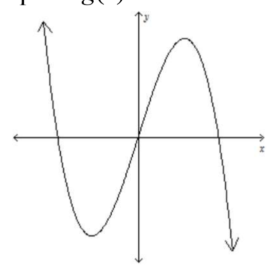
Free Response Practice #18
Calculator NOT Permitted

Pictured below are graphs of two different polynomial functions. All of the zeros of each function are real—none are imaginary. Answer the questions that follow about the two graphs, $f(x)$ and $g(x)$.

Graph of $f(x)$



Graph of $g(x)$



a. Based on the graphs, what types of polynomial functions are $f(x)$ and $g(x)$? Explain your reasoning.

+1 $f(x)$ and $g(x)$ have no imaginary roots. By the FTA, the degree of $f(x)$ and $g(x)$ will be the sum of the multiplicities of the roots.

+1
- $f(x)$ is tangent to the negative x-axis. ∴ $f(x)$ has a root multiplicity ≥ 2.
- $f(x)$ crosses the x-axis twice w/o changing concavity ∴ $f(x)$ has two roots of mult $= 1$

+1/2 ∴ $f(x)$ is even degree ≥ 4.

$g(x)$ crosses the x-axis 3 times w/o changing concavity. ∴ $g(x)$ has 3 roots of mult $= 1$. +1

∴ $g(x)$ is cubic. +1/2

b. What can be concluded about the value of a, if a is the leading coefficient in the equation of $g(x)$? Explain your reasoning.

+1
$\lim_{x \to -\infty} g(x) = \infty$
$\lim_{x \to \infty} g(x) = -\infty$

∴ $g(x)$ is odd degree with $a < 0$ +1

c. How many points of inflection does the graph of $f(x)$ have? Give a reason for your answer.

- $f(x)$ has 2 pairs of consecutive extrema.
- Points of inflection occur about halfway between consecutive extrema. +1

∴ $f(x)$ has 2 points of inflection. +1

d. If d represents the constant term in the equation of $g(x)$, what can be concluded about the value of d? Explain your reasoning.

$g(x)$ crosses the x-axis at the origin.
∴ $g(x)$ has a factor of x.
∴ $g(x)$'s constant term $d = 0$. +1

Free Response Practice #18 Grading Rubric

Free Response Part A – 4 points total

____ 1 $f(x)$ is an example of a quartic polynomial function

____ 1 The graph has a negative root of multiplicity 2 because the graph is tangent to the x – axis. The graph has two positive roots of multiplicity 1 because the graph crosses the x – axis without changing concavity. Since the sum of the multiplicities is 4, the degree is 4.

____ 1 $g(x)$ is an example of a cubic polynomial function because there are three zeros.

____ 1 The graph crosses the x – axis three times without changing concavity which means that all three zeros has a multiplicity of 1. Since the sum of the multiplicities is 3, the degree is 3.

Free Response Part B – 2 points total

____ 1 $a < 0$ because…

____ 1 As $x \to -\infty$, $g(x) \to \infty$, and as $x \to \infty$, $g(x) \to -\infty$ and the degree of the function is odd.

Free Response Part C – 2 points total

____ 1 For this function, the only points of inflection occur at midpoints between consecutive relative maximums and minimums.

____ 1 $f(x)$ has three total relative maximums and minimums so there are two points of inflection.

Free Response Part D – 1 point total

____ 1 The graph of $g(x)$ has a zero at $x = 0$ and the only way that this can occur is if x is a factor of each term in the equation which could mean that the equation of $g(x)$ has no constant term. Hence d, the constant term in the equation of $g(x)$, is 0.

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