Free Response Practice #17
Calculator Permitted

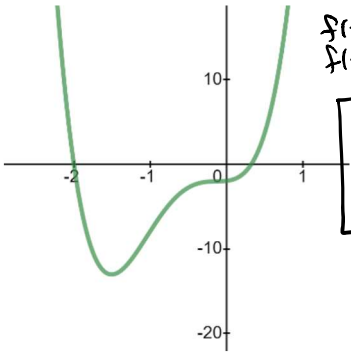
Consider the function $f(x) = 9x^4 + 21x^3 + 7x^2 + x - 2$ to answer the following questions.

a. Find $f(-2.5)$ and $f(-1.5)$. What do these values suggest about the graph of $f(x)$ on the interval $-2.5 < x < -1.5$?

$f(-2.5) = 62.688$
$f(-1.5) = -13.06$ } +1

$f(-2.5) > 0$ and $f(-1.5) < 0$.
∴ $f(x)$ changes signs between $x = -2.5$ and $x = -1.5$
∴ $f(x)$ has a root between $x = -2.5$ and $x = -1.5$ } +1

b. Use Descartes' Rule of Signs to determine the number of possible positive, negative, zero, and imaginary roots of $f(x)$. Make a chart that summarizes your results. Then, after investigating the graph of $f(x)$, which of the combinations from the table is correct and explain why.



$f(x) = 9x^4 + 21x^3 + 7x^2 + x - 2$
$f(-x) = 9x^4 - 21x^3 + 7x^2 - x - 2$

+ = 1
− = 3 or 1
zero = 0
i = 4, 2, 0 } +1

P	N	zero	i
1	3	0	0
1	1	0	2

+1

By the FTA, $f(x)$ has 4 roots.
- $f(x)$ crosses the x-axis once (w/o changing concavity) on both the negative and positive x-axis's
- $f(x)$ does not contain the origin

∴ $f(x)$ has 1 pos, 1 neg, 0 zero, and 2 imaginary roots. } +1

c. What are all of the possible rational roots of $f(x)$? Of these possible roots, which two appear to be the most likely possible roots?

$PRR = \frac{\pm 1, \pm 2}{\pm 1, \pm 3, \pm 9}$

HPRR = $-2, \frac{1}{3}$ ✓ ✓ } +1

d. Find the roots of $f(x)$, real and/or imaginary. Show all of your work.

-2	9	21	7	1	-2
		0	-18	-6	-2	2
$\frac{1}{3}$	9	3	1	-1	0
		0	3	2	1
	9	6	3	0	

+1

$f(x) = (x+2)(x-\frac{1}{3})(9x^2+6x+3)$
$f(x) = 3(x+2)(x-\frac{1}{3})(3x^2+2x+1)$

$3x^2 + 2x + 1 = 0$
$Disc = b^2 - 4ac$
$= (2)^2 - 4(3)(1)$
$= 4 - 12$
$Disc = -8$

$x = \frac{-b \pm \sqrt{Disc}}{2a}$
$x = \frac{-2 \pm \sqrt{-8}}{2(3)}$
$x = \frac{-2 \pm 2i\sqrt{2}}{6}$
$x = \frac{-1 \pm i\sqrt{2}}{3}$ } +1

+1

Roots of $f(x)$: $x = -2, \ x = \frac{1}{3}, \ x = \frac{-1-i\sqrt{2}}{3}, \ x = \frac{-1+i\sqrt{2}}{3}$

Free Response Practice #17 Grading Rubric

Free Response Part A – 2 points total

____ 1 Finds $f(-2.5) = 62.688$ and $f(-1.5) = -13.063$

____ 1 Since $f(-2.5) > 0$ and $f(-1.5) < 0$, then the graph goes from above the x – axis to below the x – axis which means that the graph must have a zero at some value on the interval $-2.5 < x < -1.5$

Free Response Part B – 3 points total

____ 1 Finds 1 sign change in the equation of $f(x)$ and three sign changes in the equation of $f(-x) = 9x^4 - 21x^3 + 7x^2 - x - 2$.

____ 1 Correct and complete chart of possible combinations of roots as shown below

Positive	Negative	Zero	Imaginary
1	3	0	0
1	1	0	2

____ 1 The graph of $f(x)$ crosses the x – axis at one negative value and one positive value. Therefore, two of the roots should be imaginary so the correct combination is 1 – 1 – 0 – 2.

Free Response Part C – 1 point total

____ 1 Possible rational roots of $f(x) = \frac{Factors\ of\ -2}{Factors\ of\ 9} = \frac{\pm1, \pm2}{\pm1, \pm3, \pm9} = \pm1, \pm\frac{1}{3}, \pm\frac{1}{9}, \pm2, \pm\frac{2}{3}, \pm\frac{2}{9}$

Free Response Part D – 3 points total

____ 1 Shows, using synthetic division, that $x = -2$ and $x = \frac{1}{3}$ are real zeros because the remainders are 0

$$\begin{array}{r|rrrr} -2 & 9 & 21 & 7 & 1 & -2 \\ & & -18 & -6 & -2 & 2 \\ \hline & 9 & -3 & 1 & -1 & \mid 0 \end{array} \qquad \begin{array}{r|rrrr} 1/3 & 9 & 3 & 1 & -1 \\ & & 3 & 2 & 1 \\ \hline & 9 & 6 & 3 & \mid 0 \end{array}$$

____ 1 Solves the equation $9x^2 + 6x + 3 = 0$ to find the two imaginary roots $x = \frac{-1 \pm i\sqrt{2}}{3}$.

$$\frac{9x^2+6x+3}{3} = \frac{0}{3} \quad \Rightarrow \quad 3x^2 + 2x + 1 = 0$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(3)(1)}}{2(3)} = \frac{-2 \pm \sqrt{-8}}{6} = \frac{-2 \pm 2i\sqrt{2}}{6} = \frac{-1 \pm i\sqrt{2}}{3}$$

____ 1 Roots of $f(x)$: $x = -2, \frac{1}{3}, -\frac{1}{3} + \frac{\sqrt{2}}{3}i$, and $-\frac{1}{3} - \frac{\sqrt{2}}{3}i$