

Free Response Practice #13

Calculator NOT Permitted

A cubic function is such that $f(-3) = 2$, $f(-1) = -3$, and $f(4) = -3$. Additionally, $f(x)$ has only two distinct zeros and they exist on the interval $-3 \leq x \leq 4$.

a. Identify the domain and range of $f(x)$.

$$\left. \begin{array}{l} D: (-\infty, \infty) \\ R: (-\infty, \infty) \end{array} \right\} \text{tl}$$

b. State the multiplicities of the two zeros of $f(x)$. Justify your reasoning.

$f(x)$ is cubic

$\therefore f(x)$ has 3 zeros

\therefore The sum of the multiplicities is 3

$f(x)$ has 2 distinct zeros, one of those zeros must have a multiplicity of 1 and the other has multiplicity of 2.

c. Explain why $f(x)$ is guaranteed to have a zero of multiplicity ~~2~~^{odd} on the interval $-3 < x < -1$.

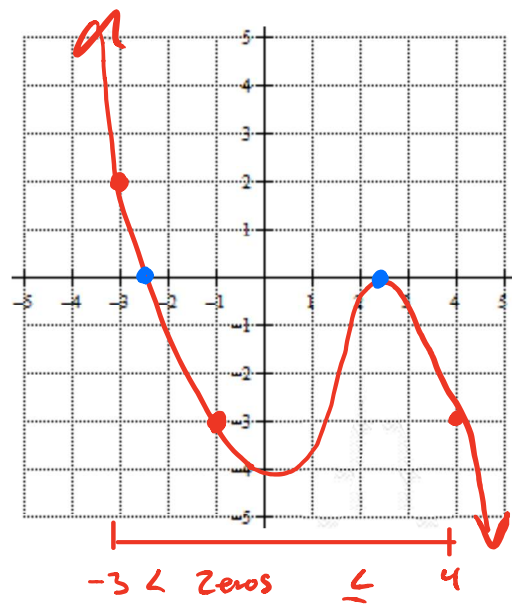
$f(-3) > 0 \therefore f(x)$ is above the x -axis at $x = -3$

$f(-1) < 0 \therefore f(x)$ is below the x -axis at $x = -1$

$\therefore f(x)$ must cross the x -axis and have an odd multiplicity of 1 on $-3 < x < -1$

d. Sketch a possible graph for $f(x)$.

See rubric for scoring



Free Response Practice #13 Grading Rubric

Free Response Part A – 1 point total

_____ 1 Domain and Range: $(-\infty, \infty)$

Free Response Part B – 2 points total

_____ 1 Since $f(x)$ is a cubic function and there are only two distinct zeros, then the sum of the multiplicities must be 3 so neither one of the zeros can have a multiplicity of 3.

_____ 1 Since there are only two distinct zeros, then one must have a multiplicity of 1 and the other has a multiplicity of 2.

Free Response Part C – 3 points total

_____ 1 $f(-3) > 0$ which means that at $x = -3$ the graph is above the x – axis.

_____ 1 $f(-1) < 0$ which means that at $x = -1$ the graph is below the x – axis.

_____ 1 Thus, at some value on the interval $-3 < x < -1$ the graph of $f(x)$ crosses the x – axis having a multiplicity of 1.

Free Response Part D – 3 points total

_____ 1 Graph contains the points $(-3, 2)$, $(-1, -3)$ and $(4, -3)$ and is drawn as a continuous, smooth curve (no sharp turns to the graph).

_____ 1 Graph crosses the x – axis without changing concavity at a value on the interval $-3 < x < -1$

_____ 1 Graph is tangent to the x – axis at some value on the interval $-1 < x < 4$

