

Free Response Practice #12

Calculator NOT Permitted

Consider the polynomial function $f(x) = x^3 + 3x^2 - 25x - 75$ to answer the following questions.

- a. Rewrite the function in completely factored form by factoring the polynomial by grouping.

$$\begin{aligned}
 f(x) &= x^3 + 3x^2 - 25x - 75 \\
 f(x) &= x^2(x+3) - 25(x+3) \\
 f(x) &= (x+3)(x^2 - 25) \\
 f(x) &= (x+3)(x-5)(x+5)
 \end{aligned}$$

- b. Use synthetic division to show that $(x+3)$ is a factor of the function. Explain why your work shows that it is a factor.

-3	1	3	-25	-75	}	+1
		-3	0	75		
	1	0	-25	0		

By the Factor Theorem, $(x+3)$ is a factor of $f(x)$ b/c $f(x)/(x+3)$ has a remainder of zero. +1

- c. Based on your result from part a), what can specifically be determined about the graph of $f(x)$ at $x = -3$? Explain your reasoning.

- $(x+3)$ is a factor of $f(x)$ once +1
- $x = -3$ is a zero of $f(x)$ with multiplicity 1 +1
- ∴ The graph of $f(x)$ crosses the x -axis at $x = -3$ and doesn't change concavity. +1

- d. Suppose that $g(x) = -3x^3 + kx - 2$. For what value of k would $(x+2)$ be a factor of $g(x)$. Show your work.

-2	-3	0	k	-2	}	+1
		6	-12	-2k+24		
	-3	6	k-12	-2k+22		

$$\begin{aligned}
 -2k+22 &= 0 \\
 -2k &= -22 \\
 k &= 11
 \end{aligned}$$