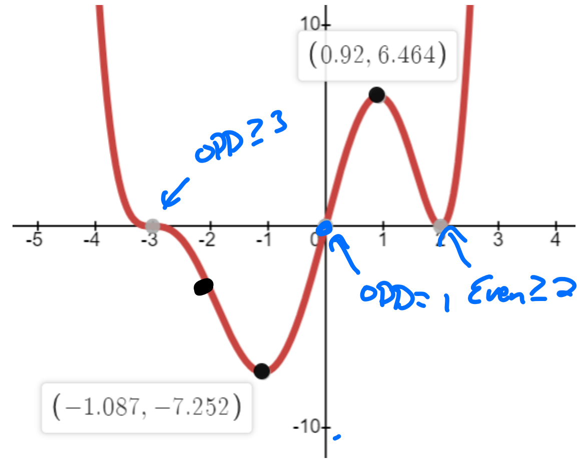
Calculator NOT Permitted

Test Review Day 1

Free Response #1

Pictured to the right is the graph of a polynomial function, $g(x)$. Use the graph to answer the questions that follow.

a. Identify the zeros of the graph of $g(x)$ and their multiplicities. Explain specifically how you know the multiplicity of each zero.



+1 • $g(x)$ crosses x-axis at $x = -3$ & changing concavity
∴ $x = 3$ is a zero (ODD mult ≥ 3)

+1 • $g(x)$ crosses x-axis at $x = 0$ w/o changing concavity
∴ $x = 1$ is a zero (ODD mult = 1)

+1 • $g(x)$ tangent x-axis at $x = 2$
∴ $x = 2$ is a zero (Even mult ≥ 2)

b. Will the leading coefficient of the equation be positive or negative? Give a reason .

$\lim_{x\to\infty} g(x) = \infty$ ∴ lead coefficient is positive

+1 +1

c. Estimate x – coordinates of points of inflection for $g(x)$. Give a reason for each of your answers or show work.

zero of odd mult ≥ 3 @ $x = -3$ ∴ $x = -3$ is point of inflection +1

PoI

$x \approx \frac{-3+(-1.087)}{2} \approx \frac{-4.087}{2} \approx -2.044$ +1

$x \approx \frac{-1.087+.92}{2} \approx \frac{-.167}{2} \approx -.084$ +1

$x \approx \frac{.92+2}{2} \approx \frac{2.92}{2} \approx 1.46$ +1

d. Identify each of the following intervals for $g(x)$.

Approximate Interval(s) where $g(x)$ is concave down: $(-3, -2.044) \cup (-.084, 1.46)$ +1

Interval(s) where $g(x) < 0$: $(-3, 0)$ +1

Multiple Choice – Calculator NOT Permitted

mult = 1 or 3

x	-4	-3.1	-3	-2.9	-2	0	1	2	3
$h(x)$	-125	-0.689	0	-0.593	-27	-9	0	25	288

Even ODD

Mult = 2 or 4 5th

1. The table above contains function values along the graph of a quintic polynomial function, $h(x)$. The only zeros of $h(x)$ are zeros specifically listed in the table. Which of the statements that follow is/are true about $h(x)$?

   I. $(x + 3)$ is a factor of at least $h(x)$ twice. ✓

   II. The multiplicity of $x = 1$ cannot be determined but it must be 1 or 3. ✓ ✓

   III. If a is the leading coefficient of the equation of $h(x)$, then $a > 0$. True

   A. I only
   B. I and II only
   C. III only
   D. I, II, and III
   E. I and III only

R = 0

2. Find the value of k so that $(x - 3)$ is a factor of $H(x) = 3x^3 - 2x^2 + kx - 3$.

   A. 34
   B. 20
   C. –20
   D. –34
   E. 32

$$\begin{array}{r|rrrr} 3 & 3 & -2 & k & -3 \\ & & 9 & 21 & (3k+63) \\ \hline & 3 & 7 & (k+21) & 3k+60 \end{array}$$

$3k + 60 = 0$
$3k = -60$
$k = -20$

3. What can be said about the value of $P(-1)$ and the remainder when $P(x) = 2x^3 - 2x^2 - x - 3$ is divided by $(x - 1)$.

   A. $P(-1)$ is greater than the remainder.
   B. $P(-1)$ is less than the remainder.
   C. $P(-1)$ is equal to the remainder.
   D. $P(-1)$ cannot be determined.
   E. None of these

$$\begin{array}{r|rrrr} -1 & 2 & -2 & -1 & -3 \\ & & -2 & 4 & -3 \\ \hline & 2 & -4 & 3 & -6 \end{array}$$

$\therefore P(-1) = -6$

$$\begin{array}{r|rrrr} 1 & 2 & -2 & -1 & -3 \\ & & 2 & 0 & -1 \\ \hline & 2 & 0 & -1 & -4 \end{array}$$

$\therefore R = -4$

4. How many times is $(x - 1)$ a factor of the polynomial function $g(x) = 2x^4 + 5x^3 - 12x^2 + x + 4$?

A. None

B. One

C. Two

D. Three

E. Four

✓ 1	2	5	−12	1	4
		2	7	−5	−4
	2	7	−5	−4	0

✓ 1	2	7	−5	−4
		2	9	4
	2	9	4	0

x 1	2	9	4
		2	11
	2	11	15

5. Which of the following binomials is/are factors of the polynomial function $f(x) = x^3 - 2x^2 - 11x + 12$?

I. $x - 1$ ✓ II. $x + 1$ x III. $x + 3$ ✓

A. I only

B. I and II only

C. I and III only

D. II only

E. III only

✓ 1	1	−2	−11	12
		1	−1	−12
	1	−1	−12	0

x −1	1	−1	−12
		−1	2
	1	−2	−10

✓ −3	1	−1	−12
		−3	12
	1	−4	0

6. If $f(x) = x^3 - 3x^2 - 13x + 15$ and $(x - 5)$ is a factor of $f(x)$, what would the function be written in completely factored form?

A. $f(x) = (x - 1)(x + 3)(x - 5)$
B. $f(x) = (x - 1)(x - 3)(x - 5)$
C. $f(x) = (x - 1)(x + 3)(x + 5)$
D. $f(x) = (x - 1)(x - 3)(x + 5)$
E. $f(x) = (x + 1)(x + 3)(x - 5)$

5	1	−3	−13	15
		5	10	−15
	1	2	−3	0

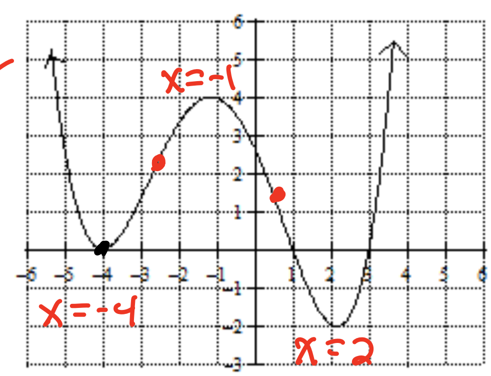
$$f(x) = (x-5)(x^2+2x-3)$$
$$= (x-5)(x+3)(x-1)$$

7. The graph to the right is a polynomial function with a relative maximum at the point (–1, 4) and relative minimums at the points (–4, 0) and (2, –2). Which of the following statements is/are true?

I. The graph changes concavity at approximately $x = -2.5$.

II. The graph has an absolute minimum at the point $(2, -2)$.

III. If c is the constant term in the equation, then c is a value between y = 2 and y = 3.



A. I only
B. I and II only
C. III only
D. I and III only
E. I, II and III

Test Review Day 2

Free Response #2

A quartic function, $h(x) = ax^4 - x^3 - 21x^2 + 41x - 20$, is represented numerically in the table of values below. $h(x)$ has three distinct zeros, one of which has a multiplicity of two. Use the equation and the table to answer the questions that follow.

x	–5.1	–5	–4.9	0.9	1	1.1	2	3	5
$h(x)$	33.861	0	-30.981	-0.183	0	-0.177	-14	-32	160

a. Based on the values in the table, is $a > 0$ or is $a < 0$? Give a reason for your answer.

$\lim_{x \to \infty} h(x) = \infty \therefore a > 0$

+1 +1

b. Use synthetic division and one of the zeros of $h(x)$ from the table, to find the value of a. Show your work.

$ax^4 - x^3 - 21x^2 + 41x - 20$

$$\begin{array}{c|ccccc} 1 & a & -1 & -21 & 41 & -20 \\ & & a & a-1 & (a-22) & (a+19) \\ \hline & a & (a-1) & (a-22) & (a+19) & \boxed{a-1} \end{array}$$

R = 0

+1

$a - 1 = 0$

$a = 1$ +1

c. Two of the zeros of $h(x)$ are specifically listed in the table. Between what two values in the table does another zero of $h(x)$ exist? Give a numerical reason for your answer.

$h(x)$ changes sign between $x = 3$ and $x = 5$ +1

$\therefore$ A zero must exist between $x = 3$ and $x = 5$. +1

d. Based on the values in the table, which zero has an even multiplicity of at least two? Give a reason for your answer.

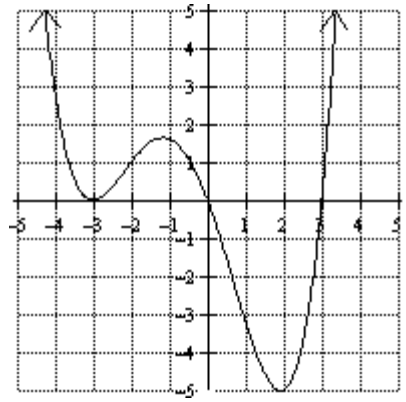
$f(x) = 0$ at $x = 1$ and on both sides of $x = 1$,
the y-values are the same sign. +1

$\therefore$ $f(x)$ is tangent to x-axis at $x = 1$ +1

$\therefore$ $x = 1$ is a zero of multiplicity ≥ 2. +1

Multiple Choice − Calculator NOT Permitted

8. Which of the following statements is/are true about the polynomial function graphed below if all of the zeros of the function are real zeros.



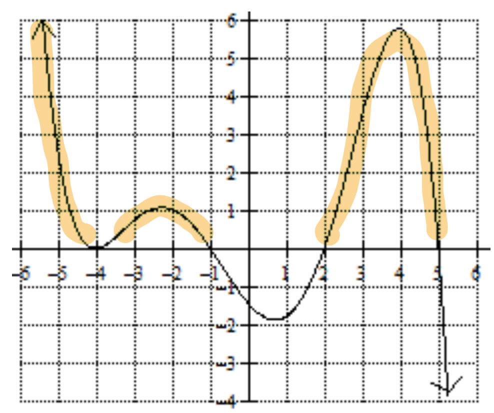
I. The negative root has even multiplicity. True

II. The function is even degree of at least 4. True

III. The positive root has a multiplicity of 1. True

A. I only
B. I and II only
C. II and III only
D. II only
E. I, II and III

9. The graph of $g(x)$ is pictured to the right. For what intervals is $g(x) > 0$?



A. $(-\infty,-1)\cup(2,5)$ B. $(-1,2)\cup(5,\infty)$

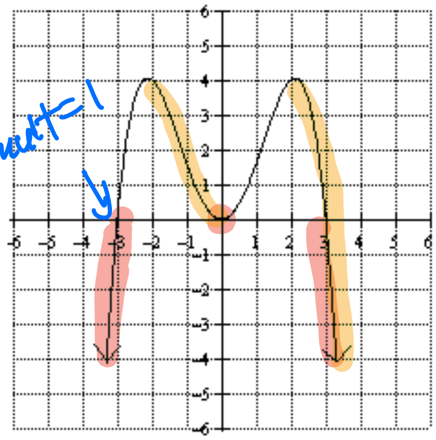
C. $(-\infty,-1]\cup[2,5]$ D. $(-\infty,-4]\cup(2,5)$

E. $(-\infty,-4)\cup(-4,-1)\cup(2,5)$

For questions 10 – 12, use the graph of the function, $G(x)$, pictured.

ODD mult=1

Graph of $G(x)$



10. Which of the following statements is/are true?

I. $G(x) \leq 0$ on the intervals $(-\infty, -3] \cup [3, \infty)$ and $x = 0$.

II. $(x + 3)$ is a factor of the equation of $G(x)$ twice. False

III. The leading coefficient of the equation of $G(x)$ is negative. True $\lim_{x \to \infty} G(x) = -\infty$

A. I only
B. III only
C. I and II only
D. I and III only
E. I, II and III

11. On which of the following intervals of x is $G(x)$ decreasing?

I. $(-\infty, -2)$ II. $(-2, 0)$ III. $(0, 2)$ IV. $(2, \infty)$

A. II and IV only
B. I and III only
C. III only
D. II and III only
E. II only

12. Which of the following statements is/are true about the graph of $G(x)$?

I. The graph is increasing on the entire interval $0 < x < 3$. False

II. The point $(2, 4)$ is an absolute maximum of the graph of $G(x)$. True

III. The graph of $G(x)$ is concave up on the entire interval $-1 < x < 1$. True

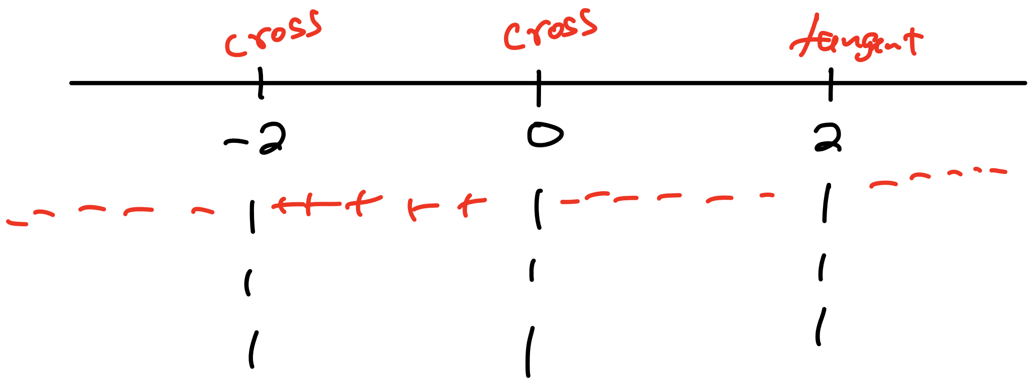
A. I and II only
B. II and III only
C. I and III only
D. III only
E. I, II, and III

13. Which of the following is/are solution interval(s) of the inequality below?

$$-2x(x+2)(x-2)^2 < 0 \quad \lim_{x \to \infty} f(x) = -\infty$$

I. $(-\infty,-2)$ ✓ II. $(0,\infty)$ ✗ III. $(-2,0)$ ✗

A. I only
B. III only
C. II only
D. I and II only
E. I and III only

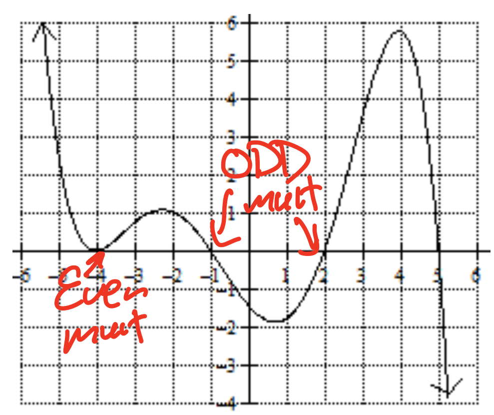


14. Which of the following statements is/are true about the function, $h(x)$, pictured to the right?

I. $(x + 4)$ is a factor of the equation of $h(x)$ at least twice. T

II. The equation of $h(x)$ has a negative leading coefficient. T

III. The graph has no points of inflection. F



A. I only
B. II and III only
C. I and II only
D. II only
E. I, II and III

1.	D
2.	C
3.	B
4.	C
5.	C
6.	A
7.	E

8.	E
9.	E
10.	D
11.	A
12.	B
13.	A
14.	C