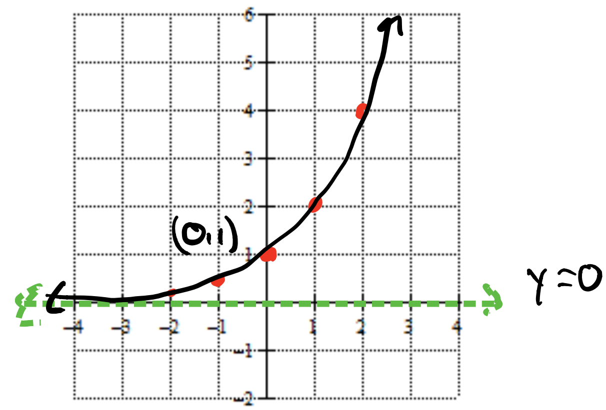
Notes 7.5 Graphing Exponential Functions

You have talked about the graphs of functions and shifting those graphs around the coordinate plane. We will now apply that to exponential functions, but first, let's talk about two different types of exponential functions—growth and decay functions.

I. Exponential Growth Functions

Consider for a moment the function $f(x) = 2^x$. Fill in the table of x and y values below and then graph them on the grid to the right.

x	$f(x) = 2^x$
–2	$2^{-2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$
–1	$2^{-1} = \left(\frac{1}{2}\right)^1 = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$

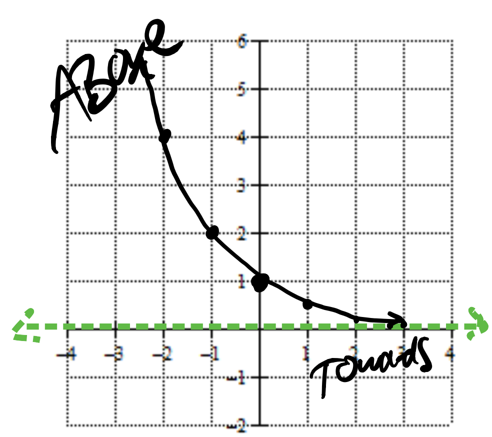


II. Exponential Decay Functions

Graph the function $f(x) = \left(\frac{1}{2}\right)^x = (2)^{-x}$ → Horizontal reflection

1. Find the location of the asymptote, giving justification.
2. State if the graph is above or below the HA, giving justification.
3. Identify if the graph is growing toward or away from HA, giving justification.
4. Use a coordinates rule to find the new location of (0, 1) on the graph as a starting point.
5. Use the base to find other points.

1. f(x) has a constant of 0
   ∴ HA @ y=0
2. a > 0 ∴ f(x) is above HA
3. c < 0 ∴ f(x) grows toward HA
4. $(x, y) \rightarrow (-x, y)$
   $(0,1) \rightarrow (0, 1)$
5.



Remember from a previous lesson, we discovered the shifting and reflecting rules for the graphs of different types of functions. Below, write the shifting rules as a review.

1. $y = -f(x)$ → vertical reflection → y-values change signs
2. $y = f(-x)$ → horizontal reflection → x-values change signs
3. $y = f(x) + k$ → vertical translation → y-values increase or decrease
4. $y = f(x-h)$ → horizontal translation → x-values increase or decrease

These shifting rules can be applied to the graphs of exponential functions as well. Let's refer back to the graph of the exponential growth function at the beginning of the lesson, $f(x) = 2^x$.

a. Describe how the graph of $g(x) = -2^{x+3} + 2$ would be different from the graph of $f(x) = 2^x$.

① vertical reflection $(a < 0)$ ∴ g(x) is below HA

② Translate left 3 & up 2

b. If you knew the basic points of the basic function $f(x) = 2^x$, how could you get points on the graph of $g(x) = -2^{x+3} + 2$

$f(x) \longrightarrow g(x)$

$(x, y) \longrightarrow (x-3, -y+2)$

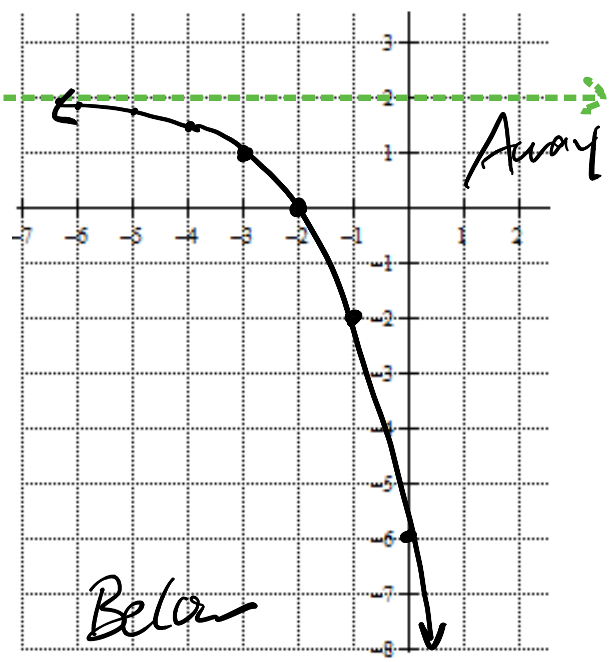
c. Graph the function $g(x) = -2^{x+3} + 2$.

1. Find the location of the asymptote, giving justification.
2. State if the graph is above or below the HA, giving justification.
3. Identify if the graph is growing toward or away from HA, giving justification.
4. Use a coordinates rule to find the new location of (0, 1) on the graph as a starting point.
5. Use the base to find other points.

1. g(x) has a constant of 2 ∴ g(x) has HA @ y = 2
2. $a < 0$ ∴ g(x) is below HA
3. $c > 0$ ∴ g(x) is going away from HA
4. $(x, y) \longrightarrow (x-3, -y+2)$

   $(0, 1) \longrightarrow (-3, 1)$

5.



d. Identify the domain and range of g(x).

D: $(-\infty, \infty)$

R: $(-\infty, 2)$

For each of the following, state the reflections and/or shifts that would be made to the graph of the mother function of each of the following. Then, state the changes that would be made to the coordinates of the mother function to obtain the graph of the given function. Then, state the domain and range of the given function.

1) $h(x) = 2^{-x-3} - 2 = 2^{-(x+3)} - 2$

- horizontal reflection & translate left 3
- vertical translate down 2.

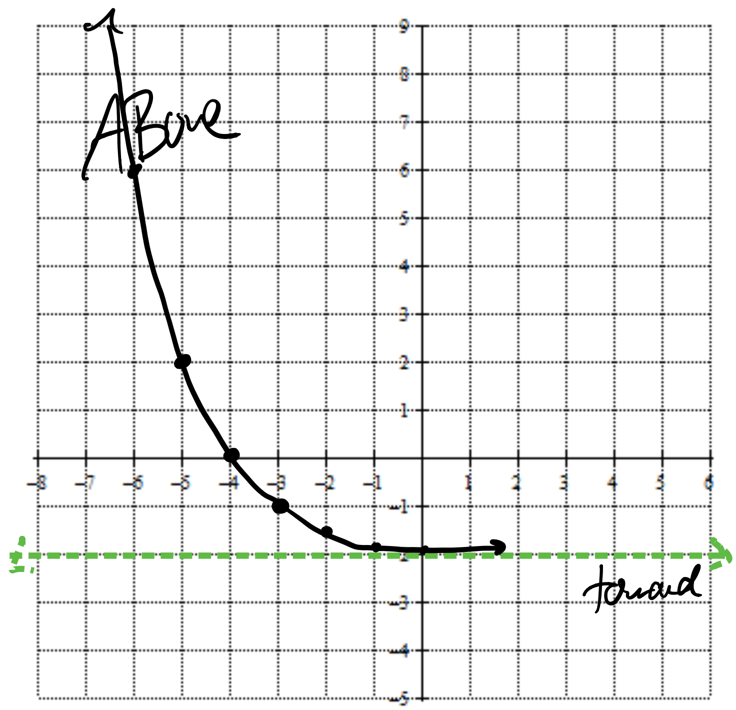
① $h(x)$ has a constant -2 $\therefore$ HA @ $y = -2$

② $a > 0$ $\therefore$ $h(x)$ is above HA

③ $c < 0$ $\therefore$ $h(x)$ is going toward HA

④ $(x, y) \longrightarrow (-x-3,\ y-2)$

$(0,1) \longrightarrow (-3,-1)$



2. $k(x) = 2^{x+2} - 5$

- translate left 2 and down 5

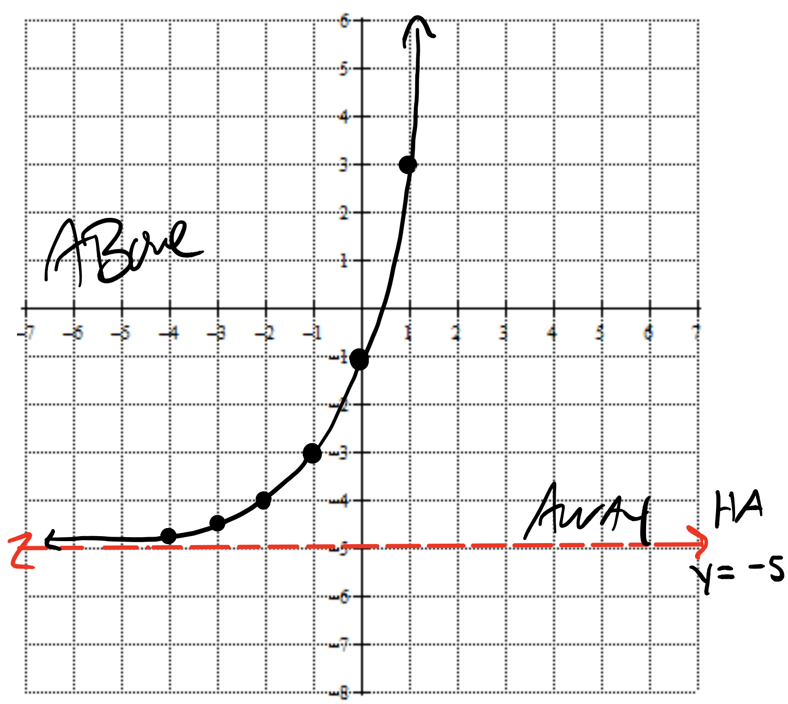
① $k(x)$ has a constant of -5 $\therefore$ HA @ $y = -5$

② $a > 0$ $\therefore$ $k(x)$ is above HA

③ $c > 0$ $\therefore$ $k(x)$ goes away from HA

④ $(x, y) \longrightarrow (x-2,\ y-5)$

$(0,1) \longrightarrow (-2,-4)$



3. $g(x) = -\left(\frac{1}{2}\right)^{x-2} + 2$ $= -(2)^{-(x-2)} + 2$

- Reflect horizontal & vertical
- Translate right 2 & up 2

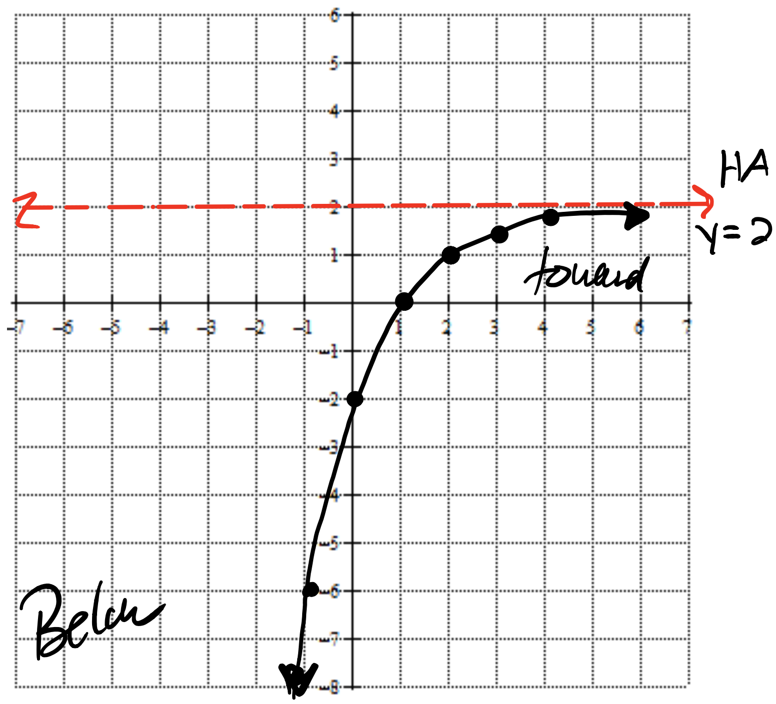
① K(x) has a constant of 2 ∴ HA @ y = 2

② a<0 ∴ K(x) is below HA

③ c<0 ∴ K(x) goes toward HA

④ $(x,y) \longrightarrow (-x+2, -y+2)$

$(x,y) \longrightarrow (2, 1)$



4. $g(x) = -\left(\frac{1}{2}\right)^{-x+3} - 1$ $= -(2)^{x-3} - 1$

- Reflect vertical
- Translate right 3 & down 1

① K(x) has a constant of -1 ∴ HA @ y = -1

② a<0 ∴ K(x) is below HA

③ c>0 ∴ K(x) goes away HA

④ $(x,y) \longrightarrow (x+3, -y-1)$

$(x,y) \longrightarrow (3, -2)$