Name____**Date**_______________________**Period**________

Homework 10.5

Solve the equations below, finding exact solutions on the interval $0 < \theta \le 2\pi$. Round your answers to the nearest thousandth of a radian, if necessary.

1. $4\,sin^2\,\theta = 3$

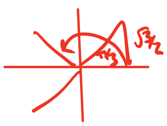
$\sin^2\theta = \frac{3}{4}$

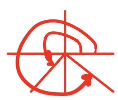
$\sin\theta = \frac{\pm\sqrt{3}}{2}$

$\sin\theta = \frac{\sqrt{3}}{2}$ or $\sin\theta = -\frac{\sqrt{3}}{2}$

$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ or $\theta = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$\theta = \pi/3, \frac{2\pi}{3}$ $\theta = \frac{4\pi}{3}, \frac{5\pi}{3}$





2. $tan\,\theta = 2\,sin\,\theta$

$\frac{\sin\theta}{\cos\theta} = 2\sin\theta$

$\sin\theta = 2\sin\theta\cos\theta$

$0 = 2\sin\theta\cos\theta - \sin\theta$

$0 = \sin\theta(2\cos\theta - 1)$

$\sin\theta = 0$

$\theta = \sin^{-1}(0)$

$\theta = \pi, 2\pi$

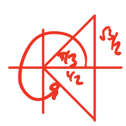
$2\cos\theta - 1 = 0$

$2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$

$\theta = \cos^{-1}\left(\frac{1}{2}\right)$

$\theta = \pi/3, 5\pi/3$



3. $1 - 3\,cos\,\theta = sin^2\,\theta$

$1 - 3\cos\theta - \sin^2\theta = 0$

$1 - 3\cos\theta - (1 - \cos^2\theta) = 0$

$\cancel{1} - 3\cos\theta \cancel{-1} + \cos^2\theta = 0$

$\cos^2\theta - 3\cos\theta = 0$

$\cos\theta(\cos\theta - 3) = 0$

$\cos\theta = 0$

$\theta = \cos^{-1}(0)$

$\theta = \pi/2, \frac{3\pi}{2}$

$\cos\theta - 3 = 0$

$\cos\theta = 3$

$\theta = \cos^{-1}(3)$

$\theta =$ undefined

4. $3\,sin\,2\theta = -\,sin\,\theta$

$3\sin 2\theta + \sin\theta = 0$

$3 \cdot 2\sin\theta\cos\theta + \sin\theta = 0$

$\sin\theta(6\cos\theta + 1) = 0$

$\sin\theta = 0$

$\theta = \sin^{-1}(0)$

$\theta = \pi, 2\pi$

$6\cos\theta + 1 = 0$

$6\cos\theta = -1$

$\cos\theta = -1/6$

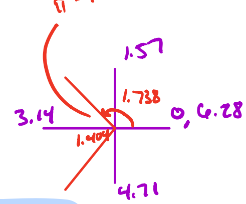
$\theta = \cos^{-1}(-1/6)$

CALCULATOR

$\theta = 1.738$

$\theta = \pi + 1.404 = 4.545$

$\pi - 1.738 = 1.404$



Solve the equations below, finding exact solutions, when possible, on the interval $0 < \theta \leq 2\pi$. Round your answers to the nearest thousandth of a radian, if necessary.

5. $4\,sin\,\theta\,cos\,\theta = \sqrt{3}$

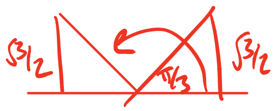
$2 \cdot 2\sin\theta\cos\theta = \sqrt{3}$

$2\sin(2\theta) = \sqrt{3}$

$\sin(2\theta) = \frac{\sqrt{3}}{2}$

$2\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$2\theta = \pi/3,\ \frac{2\pi}{3}$



$\theta = \frac{\pi}{6},\ \frac{\pi}{3},\ \frac{7\pi}{6},\ \frac{4\pi}{3}$

6. $2\,cos\,2\,\theta\,cos\,\theta + 2\,sin\,2\,\theta\,sin\,\theta = -1$

$2[\cos 2\theta\cos\theta + \sin 2\theta \sin\theta] = -1$

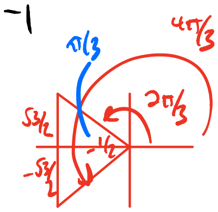
$2 \cdot [\cos(2\theta - \theta)] = -1$

$2\cos\theta = -1$

$\cos\theta = -1/2$

$\theta = \cos^{-1}(-1/2)$

$\theta = \frac{2\pi}{3},\ \frac{4\pi}{3}$



Remember, you can check your solutions to #1 – 6 by graphing each side of the equation and finding the intersection of the two graphs.

Find $\sin\theta = ?$

7. If $sin(\pi + \theta) = -\frac{3}{5}$, what is the value of $csc^2\,\theta$?

$\sin\pi\cos\theta + \sin\theta\cos\pi = -3/5$

$0 \cdot \cos\theta + \sin\theta \cdot (-1) = -3/5$

$-\sin\theta = -3/5$

$\sin\theta = 3/5$

$\csc\theta = 5/3$

$\csc^2\theta = 25/9$

8. If $cos\left(\frac{\pi}{4} + \theta\right) = -\frac{6}{7}$, find the value of $cos\,\theta - sin\,\theta$.

$\cos\frac{\pi}{4}\cos\theta - \sin\frac{\pi}{4}\sin\theta = -\frac{6}{7}$

$\frac{\sqrt{2}}{2}\cos\theta - \frac{\sqrt{2}}{2}\sin\theta = -\frac{6}{7}$

$\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{2}(\cos\theta - \sin\theta) = -\frac{6}{7} \cdot \frac{2}{\sqrt{2}}$

$\cos\theta - \sin\theta = \frac{-12}{7\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-12\sqrt{2}}{7 \cdot 2} = \frac{-6\sqrt{2}}{7}$

$\cos\theta - \sin\theta = \frac{-6\sqrt{2}}{7}$

9. If $cos\left(\frac{\pi}{4} - \theta\right) = \frac{2}{3}$, then what is the exact value of $(cos\,\theta + sin\,\theta)$?

$\cos\frac{\pi}{4}\cos\theta + \sin\frac{\pi}{4}\sin\theta = 2/3$

$\frac{\sqrt{2}}{2}\cos\theta + \frac{\sqrt{2}}{2}\sin\theta = 2/3$

$\frac{\sqrt{2}}{2}(\cos\theta + \sin\theta) = 2/3$

$\cos\theta + \sin\theta = \frac{2}{3}\cdot\frac{2}{\sqrt{2}}$

$\cos\theta + \sin\theta = \frac{4}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{3\cdot 2} = \frac{2\sqrt{2}}{3}$

$\cos\theta + \sin\theta = \frac{2\sqrt{2}}{3}$

β in II, III   II β in II, IV

4, 5, -3 $\tan\beta = \frac{4}{-3}$

10. If $\boxed{cos\,\beta = -\frac{3}{5}}$ and $\boxed{tan\,\beta < 0}$, what is the exact value of $tan\left(\frac{3\pi}{4} - \beta\right)$.

$\tan\left(\frac{3\pi}{4} - \beta\right) = \frac{\tan\left(\frac{3\pi}{4}\right) - \tan\beta}{1 + \tan\left(\frac{3\pi}{4}\right)\tan\beta}$

$= \frac{-1 - (-4/3)}{1 + (-1)(-4/3)}$

$= \frac{-1 + 4/3}{1 + 4/3} \cdot \frac{3}{3}$

$= \frac{-3 + 4}{3 + 4}$

$\tan\left(\frac{3\pi}{4} - \beta\right) = \frac{1}{7}$

11. If $f(\theta) = sin\,\theta\,cos\,\theta$ and $g(\theta) = cos^2\,\theta$, for what exact value(s) of θ on $0 < \theta \leq \pi$ does $f(\theta) = g(\theta)$?

$f(\theta) = g(\theta)$

$\sin\theta\cos\theta = \cos^2\theta$

$\sin\theta\cos\theta - \cos^2\theta = 0$

$\cos\theta(\sin\theta - \cos\theta) = 0$

$\cos\theta = 0$ $\sin\theta - \cos\theta = 0$

$\theta = \cos^{-1}(0)$ $\sin\theta = \cos\theta$

$\theta = \pi/2$ $\theta = \pi/4$

12. Sketch a graph of $f(\vartheta)$ and $g(\vartheta)$ on the axes below. Then, graphically find the intersection of the two functions. How does this graph verify or contradict your answer(s) to question 11?

$f(\theta) = \sin\theta\cos\theta$ $g(\theta) = \cos^2\theta$