

Name \_\_\_\_\_

Date \_\_\_\_\_

Period \_\_\_\_\_

0, 30°, 45°, 60°, 90°, 120°, 150°...  
45°, 90°, 135°, 180°, 225°

Homework 10.3

For problems 1 - 4, use a sum or difference identity to find the exact value of each expression.

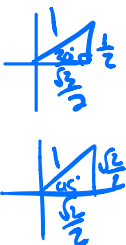
1.  $\sin(-15^\circ) = \sin(30^\circ - 45^\circ)$

$$= \sin 30^\circ \cos 45^\circ - \sin 45^\circ \cos 30^\circ$$

$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$= \frac{\sqrt{2} - \sqrt{6}}{4}$$



2.  $\sin\left(\frac{11\pi}{12}\right) = \sin\left(\frac{9\pi}{12} + \frac{2\pi}{12}\right) = \sin\left(\frac{3\pi}{4} + \frac{\pi}{6}\right)$

$$= \sin\left(\frac{3\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{6}\right)\cos\left(\frac{3\pi}{4}\right)$$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$



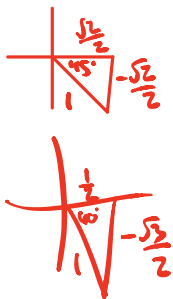
3.  $\cos(-105^\circ) = \cos(-45^\circ - 60^\circ)$

$$= \cos(-45^\circ)\cos(60^\circ) - \sin(-45^\circ)\sin(60^\circ)$$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$= \frac{\sqrt{2} - \sqrt{6}}{4}$$



4.  $\cos(75^\circ) = \cos(45^\circ + 30^\circ)$

$$= \cos(45^\circ)\cos(30^\circ) - \sin(45^\circ)\sin(30^\circ)$$

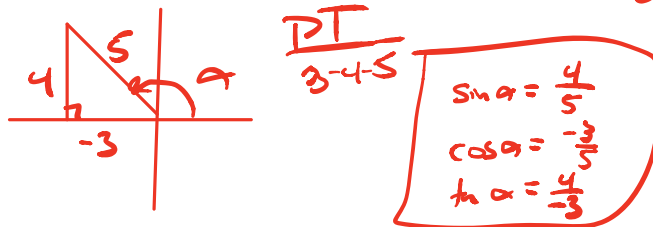
$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

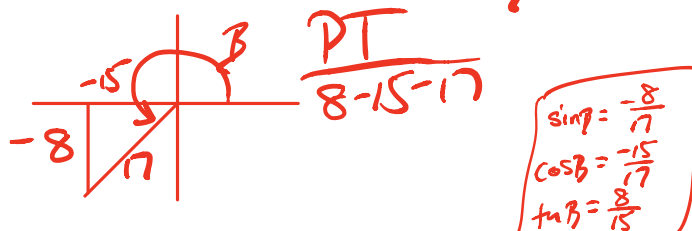
$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

For problems 5 - 9, suppose  $\cos \alpha = -\frac{3}{5}$  and  $\cos \beta = -\frac{15}{17}$ . Additionally,  $\frac{\pi}{2} < \alpha < \pi$  and  $\pi < \beta < \frac{3\pi}{2}$ .

5. Draw and label the reference triangle for  $\alpha$ .



6. Draw and label the reference triangle for  $\beta$ .



7.  $\sin(\beta - \alpha) = \sin \beta \cos \alpha - \sin \alpha \cos \beta$

$$= \left(-\frac{8}{17}\right)\left(-\frac{3}{5}\right) - \left(\frac{4}{5}\right)\left(-\frac{15}{17}\right)$$

$$= \frac{24}{85} + \frac{60}{85}$$

$$= \frac{84}{85}$$

8.  $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$

$$= \left(-\frac{3}{5}\right)\left(-\frac{15}{17}\right) + \left(\frac{4}{5}\right)\left(-\frac{8}{17}\right)$$

$$= \frac{45}{85} - \frac{32}{85}$$

$$= \frac{13}{85}$$

9.  $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$

$$= \frac{\frac{-4}{3} + \frac{8}{15}}{1 - \left(-\frac{4}{3}\right)\left(\frac{8}{15}\right)}$$

$$= \frac{-\frac{20}{15} + \frac{8}{15}}{1 - \frac{-32}{45}}$$

$$= \frac{-\frac{12}{15}}{\frac{45}{45} + \frac{32}{45}}$$

$$= \frac{-\frac{12}{15}}{\frac{77}{45}}$$

$$= \frac{-36}{77}$$

Simplify each of the following expressions using a sum or difference identity. Show your work providing exact values, when applicable. (NO CALCULATOR)

$$\begin{aligned}
 10. \sin\left(x + \frac{\pi}{2}\right) &= \sin x \cos \frac{\pi}{2} + \sin \frac{\pi}{2} \cos x \\
 &= \sin x \cdot (0) + (1) \cos x \\
 &= 0 + \cos x \\
 &= \cos x
 \end{aligned}$$

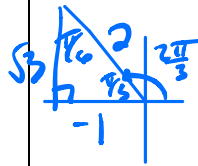
$$\begin{aligned}
 11. \cos \frac{11\pi}{9} \cos \frac{2\pi}{9} + \sin \frac{11\pi}{9} \sin \frac{2\pi}{9} \\
 &= \cos\left(\frac{11\pi}{9} - \frac{2\pi}{9}\right) \\
 &= \cos\left(\frac{9\pi}{9}\right) \\
 &= \cos(\pi) \\
 &= -1
 \end{aligned}$$

$$\begin{aligned}
 12. \cos\left(\frac{\pi}{20}\right) \cos\left(\frac{\pi}{5}\right) - \sin\left(\frac{\pi}{20}\right) \sin\left(\frac{\pi}{5}\right) \\
 &= \cos\left(\frac{\pi}{20} + \frac{\pi}{5}\right) \\
 &= \cos\left(\frac{\pi}{20} + \frac{4\pi}{20}\right) \\
 &= \cos\left(\frac{5\pi}{20}\right) \\
 &= \cos\left(\frac{\pi}{4}\right) \\
 &= \frac{\sqrt{2}}{2}
 \end{aligned}$$

$$13. \sin 400^\circ \cos 85^\circ - \cos 400^\circ \sin 85^\circ$$

$$\begin{aligned}
 &= \sin(400^\circ - 85^\circ) \\
 &= \sin(315^\circ) \\
 &= -\frac{\sqrt{2}}{2}
 \end{aligned}$$

$$\begin{aligned}
 14. \frac{\tan\left(\frac{11\pi}{21}\right) + \tan\left(\frac{\pi}{7}\right)}{1 - \tan\left(\frac{11\pi}{21}\right) \tan\left(\frac{\pi}{7}\right)} &= \tan\left(\frac{11\pi}{21} + \frac{\pi}{7}\right) \\
 &= \tan\left(\frac{11\pi}{21} + \frac{3\pi}{21}\right) \\
 &= \tan\left(\frac{14\pi}{21}\right) \\
 &= \tan\left(\frac{2\pi}{3}\right) \\
 &= -\sqrt{3}
 \end{aligned}$$



$$15. \sin(\pi + x) + \sin(\pi - x)$$

$$\begin{aligned}
 &= \sin \pi \cos x + \sin x \cos \pi + \sin \pi \cos x - \sin x \cos \pi \\
 &= 2 \sin \pi \cos x \\
 &= 2(0) \cos x \\
 &= 0
 \end{aligned}$$

