Free Response Question 4
Calculator NOT Permitted

Consider the two logarithmic functions below to answer the following questions.

$f(x) = 2\log_3 x + \log_3(x-2)$ (x>0, x>2) $\qquad g(x) = \log_3(3x)$ (x>0)

a. Rewrite $f(x)$ as a logarithm function of a single logarithm. Then, find $f(2)$ and $f(3)$. If a value is undefined, then explain why. Show your work.

$f(x) = \log_3 x^2 + \log_3(x-2)$

$f(x) = \log_3(x^3 - 2x^2)$ +1

$f(2) = \log_3(2^3 - 2(2)^2) = \log_3(8-8) = \log_3 0 =$ No solution +1

b/c the argument must be > 0. +1

$f(3) = \log_3(3^3 - 2(3)^2) = \log_3(27 - 2(9)) = \log_3(27-18) = \log_3 9 = 2$ +1

b. Between what two integers does the value of $g(20)$ lie? Show and explain your reasoning.

$g(x) = \log_3(3\cdot 20) = \log_3(60)$ +1

The argument, 60, falls between $3^3 = 27$ and $3^4 = 81$, +1

$3 < \log_3(60) < 4$

c. For what value(s) of x is $f(x) = g(x)$? Show the algebraic analysis that leads to your answer(s).

$f(x) = g(x)$

+1 $\log_3(x^3 - 2x^2) = \log_3(3x)$

$x^3 - 2x^2 = 3x$

$x^3 - 2x^2 - 3x = 0$

$x(x^2 - 2x - 3) = 0$

$x(x-3)(x+1) = 0$

$x = 0,\ x = 3,\ x = -1$ +1

$x = 0$ and $x = -1$ make both $g(x)$ and $f(x)$ undefined b/c the argument must be positive. +1

$\therefore x = 3$