Free Response Question 3
Calculator Permitted

The relationship between the intensity, I, of light, measured in lumens, at a depth of x feet in Lake Erie is given by the function below.

$$\log\left(\frac{I}{12}\right) = -0.00235x$$

The relationship between the intensity, I, of light, measured in lumens, at a depth of x feet in Lake Superior is given by the function below.

$$\log\left(\frac{I}{12}\right) = -0.0125x$$

a. What is the intensity of the light in Lake Erie at a depth of 70 feet? Show your work and indicate units of measure.

$$\log\left(\frac{I}{12}\right) = -0.00235(70) \quad \text{+1}$$

(Log Form) $\quad \log\left(\frac{I}{12}\right) = -0.1645$

(Exp Form) $\quad 10^{-0.1645} = \frac{I}{12} \quad \text{+1}$

$$12 \cdot 10^{-0.1645} = I$$

$$8.216 \approx I$$

The intensity of light in Lake Erie at 70 feet is 8.216 lumens +1

b. At what depth, to the nearest foot, is the intensity of light in Lake Superior 2.8 lumens? Show your work and indicate correct units of measure.

$$\log\left(\frac{28}{12}\right) = -0.0125x \quad \text{+1}$$

$$\frac{\log\left(\frac{2.8}{12}\right)}{-0.0125} = x$$

$$50.562 \approx x \quad \text{+1}$$

The intensity of light in Lake Superior is 2.8 lumens when the depth is about 51 feet. +1

c. Solve both intensity equations for I in terms of x. By investigating graphically, at what depth, for $x > 0$, is the intensity in both lakes the same. Show your work and explain your reasoning.

$$\log\left(\frac{I}{12}\right) = -0.00235x$$

$$10^{-0.00235x} = \frac{I}{12}$$

$$12 \cdot (10)^{-0.00235x} = I \quad \text{+1}$$

$$\log\left(\frac{I}{12}\right) = -0.0125x$$

$$10^{-0.0125x} = \frac{I}{12}$$

$$12(10)^{-0.0125x} = I \quad \text{+1}$$

$$Y_1 = 12 \cdot (10)^{-0.00235x}$$

$$Y_2 = 12(10)^{-0.0125x}$$

The graphs only intersect at $x=0$, therefore there is no depth for which the intensities of light are the same. +1