Free Response Question 2
Calculator Permitted

Consider the two logarithm functions below to answer the questions that follow. If ever the value of a logarithm is undefined, explain why it is so.

$x>5$ $\quad$ $x<2/3$

$f(x) = \log_3(x-5) + \log_3 x \qquad g(x) = \log_3(6-4x)$

a. Rewrite *f*(*x*) as a logarithm function of a single logarithm. Then, find the value of *f*(7).

$f(x) = \log_3(x-5) + \log_3 x$

+1 $f(x) = \log_3(x^2-5x)$

$f(7) = \log_3[(7)^2-5(7)]$

$= \log_3[49-35]$

$= \log_3[14]$ +1

$f(7) = \frac{\log 14}{\log 3}$

$f(7) \approx 2.402$ +1

b. If *h*(*x*) = *g*(*x*) – *f*(*x*), find an equation for *h*(*x*) that contains a single logarithm and then find the value of *h*(1).

$h(x) = g(x) - f(x)$

$h(x) = \log_3(6-4x) - \log_3(x^2-5x)$

$h(x) = \log_3 \frac{6-4x}{x^2-5x}$ +1

$h(1) = \log_3 \frac{6-4(1)}{(1)^2-5(1)}$

$= \log_3 \frac{6-4}{1-5}$

$= \log_3\left(\frac{2}{-4}\right)$

$h(1) = \log_3\left(-\frac{1}{2}\right)$ +1

+1 h(1) is undefined because the argument must be > 0 to be defined.

c. For what value(s) of *x* is *f*(*x*) = *g*(*x*)? Show the algebraic analysis that leads to your answer.

$g(x) = f(x)$

$\log_3(6-4x) = \log_3(x^2-5x)$ +1

$6-4x = x^2-5x$

$0 = x^2-x-6$

$0 = (x-3)(x+2)$

$x \neq 3, \quad x \neq -2$ +1

g(x) = f(x) has no solution because x = 3 makes f(x) undefined and x = -2 makes g(x) undefined. +1