Free Response Question 1
Calculator Permitted

A colony of ladybugs rapidly multiplies so that the population t days from now is given by the equation

$$A(t) = 3000e^{0.01t}.$$

Similarly, a pesticide is added to another colony of ladybugs so that the population t days from now is given by the equation

$$D(t) = 4500\left(\frac{2}{3}\right)^{0.07t}.$$

a. Find the initial population of each colony of ladybugs. Explain your work.

+1

Colony $A(t)$: $A(0) = 3000e^{0.01(0)} = 3000e^0 = 3000(1) = 3000$ lady bugs

Colony $D(t)$: $D(t) = 4500\left(\frac{2}{3}\right)^{0.07(0)} = 4500\left(\frac{2}{3}\right)^0 = 4500(1) = 4500$ ladybugs

The initial population is found when $t=0$ b/c t represents the # days from now

+1

b. During what day will the growing colony of ladybugs reach a population of 4000 ladybugs? Show your work.

$4000 = 3000e^{0.01t}$

$\frac{4}{3} = e^{0.0t}$

$\ln\left(\frac{4}{3}\right) = \ln e^{0.01t}$

$\ln\left(\frac{4}{3}\right) = 0.01t$

$100\ln\left(\frac{4}{3}\right) = t$

$28.768 \approx t$

+1

The growing colony's population will reach 4000 ladybugs during day 28. +1

c. During what day will the pesticide treated colony be halved? Show your work.

+1

$\frac{1}{2}(4500) = 4500\left(\frac{2}{3}\right)^{0.07t}$

$\frac{1}{2} = \left(\frac{2}{3}\right)^{0.07t}$

$\ln\left(\frac{1}{2}\right) = \ln\left(\frac{2}{3}\right)^{0.07t}$

$\ln\left(\frac{1}{2}\right) = 0.07t \cdot \ln\left(\frac{2}{3}\right)$

$\frac{\ln\left(\frac{1}{2}\right)}{\ln(2/3)} = 0.07t$

$\frac{100}{7} \cdot \frac{\ln\left(\frac{1}{2}\right)}{\ln(2/3)} = t$

$24.422 \approx t$ +1

The pestecide treated colony will be halved during the 24th day. +1

d. After how many days will the population of each colony have the same number lady bugs? Explain how you determined your answer based on the graphs of each function.

Each colony will have the same population when the graphs of $A(t)$ and $D(t)$ intersect +1

$A(t) = D(t)$

$3000e^{0.01t} = 4500\left(\frac{2}{3}\right)^{0.07t}$

$t = 10.564$ days

+1