Homework 4.3

For problems 1 – 4, find each of the indicated graphical properties. If a function does not have a particular property, explain why it does not. Show your work or explain your reasoning.

Zero @ $x=-4/3$ y-int $= \frac{4}{-4}$

1. $f(x) = \frac{(3x+4)\cancel{(x+2)}}{(x-4)\cancel{(x+2)}} = \frac{3x+4}{x-4}$, hole $(-2, \frac{1}{3})$

VA @ $x=4$ Hole @ $x=-2$

(a) Zero(s): $(-\frac{4}{3}, 0)$

$3x+4=0$
$3x=-4$
$x=-4/3$

(b) y – intercept: $(0,-1)$

y-int $= \frac{4}{-4} = -1$

(c) Vertical Asymptote(s):

$x-4=0$
$x=4$

(d) Coordinates of hole(s):

$x+2=0$
$x=-2$

$y=\frac{3x+4}{x-4}$
$y=\frac{3(-2)+4}{(-2)-4}$
$y=\frac{-6+4}{-6}$
$y=\frac{-2}{-6}$
$y=\frac{1}{3}$

$(-2, \frac{1}{3})$

y-int $=-1$ zero

2. $p(x) = \frac{x^2+x-6}{x^2+5x+6} = \frac{\cancel{(x+3)}(x-2)}{\cancel{(x+3)}(x+2)}$

Hole VA

(a) Zero(s): $(2,0)$

$x-2=0$
$x=2$

(b) y – intercept: $(0,-1)$

y-int $= \frac{-6}{6} = -1$

(c) Vertical Asymptote(s):

$x+2=0$
$x=-2$

(d) Coordinates of hole(s):

$x+3=0$
$x=-3$

$p(x)=\frac{x-2}{x+2}$
$p(-3)=\frac{(-3)-2}{(-3)+2}$
$=\frac{-5}{-1}$
$p(-3)=5$

$(-3,5)$

y-int $=3$

3. $h(x) = \frac{2x-6}{x^2-x-2} = \frac{2(x-3)}{(x-2)(x+1)}$ zero

VA VA

(a) Zero(s): $(3,0)$

$x-3=0$
$x=3$

(b) y – intercept: $(0,3)$

y-int $= \frac{-6}{-2} = 3$

(c) Vertical Asymptote(s):

$x-2=0$
$x=2$

$x+1=0$
$x=-1$

(d) Coordinates of hole(s):

h(x) does not have any holes because there are no canceling factors in the ratio.

zero $(-3,5)$

4. $g(x) = \frac{2x^2-5x+2}{x^2-4} = \frac{(2x-1)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}}$ Hole

y-int $=-\frac{1}{2}$ VA

(a) Zero(s):

$2x-1=0$
$2x=1$
$x=1/2$

(b) y – intercept:

y-int $= \frac{2}{-4} = -\frac{1}{2}$

(c) Vertical Asymptote(s):

$x+2=0$
$x=-2$

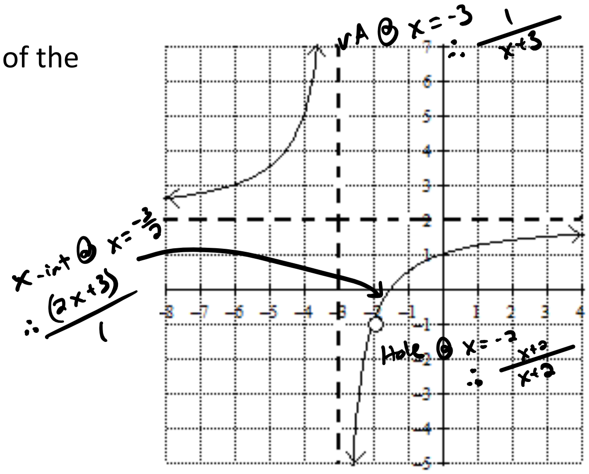
(d) Coordinates of hole(s):

$x-2=0$
$x=2$

$g(x)=\frac{2x-1}{x+2}$
$g(2)=\frac{2(2)-1}{(2)+2}$
$=\frac{4-1}{4}$
$g(2)=3/4$

$(2, 3/4)$

5. In both factored and standard form, what is an equation of the rational function, $h(x)$, pictured to the right?

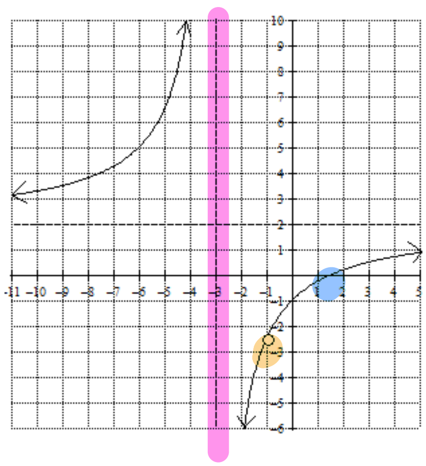


$$h(x) = \frac{(2x+3)(x+2)}{(x+3)(x+2)}$$

$$h(x) = \frac{2x^2+7x+6}{x^2+5x+6}$$

Is the y-int correct? If NOT, mult numerator by a number to fix it.

The graph of a rational function, $g(x)$, is pictured below. Answer the questions that follow.



6. What factor(s) is/are guaranteed to be in the denominator of the equation of $g(x)$? Justify your answer.

$g(x)$ is undefined at $x=-3$ and $x=-1$

∴ $g(x)$ is guaranteed to have a factor of $(x+3)$ and $(x+1)$ in the denominator.

7. What factor is guaranteed to be in both the numerator and denominator of the equation of $g(x)$? Justify your answer.

$g(x)$ has a hole at $x=-1$

∴ $g(x)$ is guaranteed to have a factor of $(x+1)$ in both the numerator and denominator.

8. What factor is guaranteed to be in the denominator of the equation of $g(x)$ but not in the numerator? Justify your answer.

$g(x)$ has a vertical asymptote at $x=-3$

∴ $g(x)$ is guaranteed to have a factor of $(x+3)$ in the denominator

9. If $g\left(\frac{3}{2}\right) = 0$, then what is the equation of $g(x)$ in both factored and standard form?

$$g(x) = \frac{(2x-3)(x+1)}{(x+3)(x+1)}$$

$$g(x) = \frac{2x^2-x-3}{x^2+4x+3}$$

Is the y-int correct? If NOT, mult numerator by a number to fix it.

10. What are the domain and range of $g(x)$?

Domain: $(-\infty,-3)\cup(-3,-1)\cup(-1,\infty)$

Range: $(-\infty,-5/2)\cup(-5/2,2)\cup(2,\infty)$