Calculator NOT Permitted

Multiple Choice		
Free Response		
Total out of 18 points		

Multiple Choice

1.	A
2.	D
3.	A
4.	D
5.	E
6.	C
7.	B
8.	C
9.	C

This reviews the second half of unit 1. Please also study quiz #1 review, each FRQ from the homework, and read over your notes.

FREE RESPONSE

Consider the two piece-wise defined functions, $f(x)$ and $g(x)$, below to answer the questions that follow.

$$f(x)=\begin{cases}\frac{1}{2}x^2-x+2, & -4<x\le -2\\ \sqrt{x+6}+4, & x>-2\end{cases} \qquad g(x)=\begin{cases}\frac{1}{2}x+4, & x<-4\\ -\frac{1}{2}x, & x>-4\end{cases}$$

a. Using interval notation, identify the domain of both functions.

Domain of $f(x)$: $(-4,\infty)$ Domain of $g(x)$: $(-\infty,-4)\cup(-4,\infty)$

+1/2 +1/2

b. Find the values of $f(-2)$ and $g(f(3))$. Show the analysis that leads to your answers.

$f(-2)=\frac{1}{2}(-2)^2-(-2)+2$
$=\frac{1}{2}(4)+2+2$
$=2+2+2$
$f(-2)=6$ +1

$f(3)=\sqrt{(3)+6}+4$
$=\sqrt{9}+4$
$=3+4$
$f(3)=7$ +1

$g(f(3))=g(7)$
$=-\frac{1}{2}(7)$
$g(f(3))=-\frac{7}{2}$ +1

c. Does $f(x)$ have a discontinuity at $x = -2$? If so, classify the discontinuity, justifying your conclusion.

I. $f(-2) = 6$ +1

$\therefore f(-2)$ is defined.

II. $\lim_{x \to -2^-} f(x) = \frac{1}{2}(-2)^2 - (-2) + 2 = \frac{1}{2}(4) + 2 + 2 = 2 + 2 + 2 = 6$

$\lim_{x \to -2^+} f(x) = \sqrt{(-2)+6} + 4 = \sqrt{4} + 4 = 2 + 4 = 6$

$\therefore \lim_{x \to -2} f(x)$ exists +1

III $\lim f(x) = f(-2) = 6$ +1

$\therefore f(x)$ is continuous at $x = -2$ +1

d. Does $g(x)$ have a discontinuity at $x = -4$? If so, classify the discontinuity, justifying your conclusion.

I. $g(-4)$ is undefined +1

II. $\lim_{x \to -4^-} g(x) = \frac{1}{2}(-4) + 4 = -2 + 4 = 2$

$\lim_{x \to -4^+} g(x) = -\frac{1}{2}(-4) = 2$

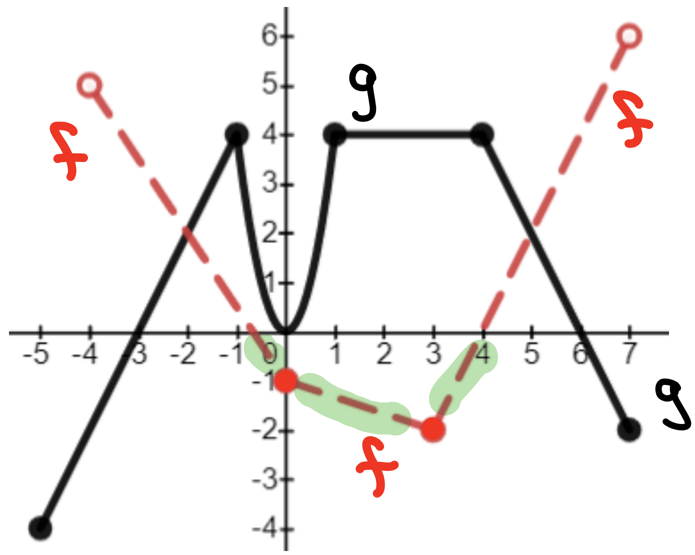
$\lim_{x \to -4} g(x)$ exists +1

III $g(-4) \neq \lim_{x \to -4} g(x)$ +1

$\therefore g(x)$ has point discontinuity at $x = -4$ +1

MULTIPLE CHOICE

Use the graph to answer questions 1 and 2. The dashed graph is $f(x)$ and the solid graph is $g(x)$.



1. At which of the following values of x is $f(x) < 0$ and $f(x) < g(x)$?

   I. $x = -1$ II. $x = 2$ III. $x = 4$

   f < 0 ✗ f < g ✓ f < 0 ✓ f < g ✓ f < 0 ✗ f < g ✓

   A. II only
   B. I and II only
   C. III only
   D. II and III only
   E. I, II, and III

   f(x) is below x-axis

   f(x) is below g(x)

2. Which of the following best describes where the graph of $g(x) \leq 0$?

   A. $[-5,-3]\cup[6,7]$ B. $(-3,0)\cup(0,6)$ ✗ C. $(-5,-3]\cup[6,7)$ ✗

   D. $[-5,-3]\cup[6,7]$ and $x = 0$ ✓ ✓ ✓ E. $[-3,6]$ ✗

3. Consider the functions $f(x) = 2x^2 + 3x - 2$ and $g(x) = x - 2$. Find an equation for $(f \cdot g)(x)$.

A. $(f \cdot g)(x) = 2x^3 - x^2 - 8x + 4$

B. $(f \cdot g)(x) = 2x^3 - 4x^2 - 5x + 4$

C. $(f \cdot g)(x) = 2x^2 - 5x$

D. $(f \cdot g)(x) = x^2 - 8x + 4$

E. None of these

$(2x^2 + 3x - 2)(x - 2)$

$= 2x^3 + 3x^2 - 2x$
$\quad - 4x^2 - 6x + 4$

$2x^3 - x^2 - 8x + 4$

4. Identify the domain of the function $g(x) = \dfrac{3-x}{x^2 - x - 20}$

A. $(-\infty,-4)\cup(-4,3)\cup(3,5)\cup(5,\infty)$

B. $(-\infty,3)\cup(3,\infty)$

C. $(-\infty,\infty)$

D. $(-\infty,-4)\cup(-4,5)\cup(5,\infty)$

E. The domain cannot be determined.

Denom ≠ 0
$(x-5)(x+4) \neq 0$
$x-5 \neq 0$, $x+4 \neq 0$
$x \neq 5$, $x \neq -4$

5. The graph of $f(x)$ is shown to the right and $g(x) = 5 - 2x$. What is the value of $f(g(5))$?

A. –3

B. 7

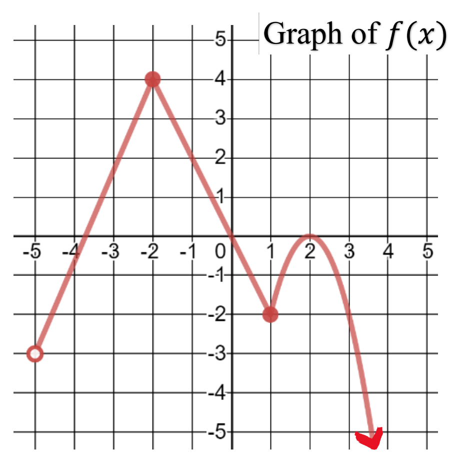
C. 19

D. –2

E. Undefined

$g(5) = 5 - 2(5) = -5$

$f(-5)$ is undefined



6. What is the domain of the function $f(x) = \sqrt{9 - 3x}$.

A. $[3,\infty)$

B. $(-\infty,3)$

C. $(-\infty,3]$

D. $(-\infty,3)\cup(3,\infty)$

E. $(3,\infty)$

RADICAND ≥ 0
$9 - 3x \geq 0$
$-3x \geq -9$
$x \leq 3$

7. For what value of a would the function $g(x)=\begin{cases} ax-3, & x<-2 \\ x^2-2x, & x>-2 \end{cases}$ have a point discontinuity at $x = -2$.

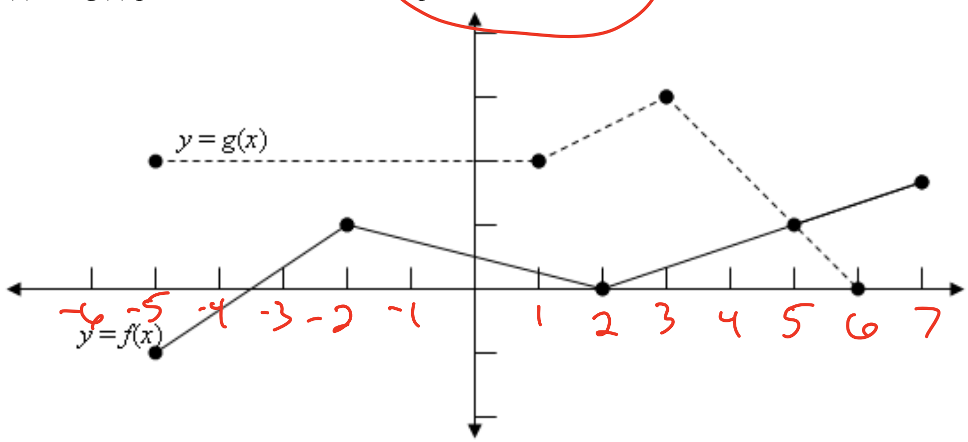
$$\lim_{x\to -2-}(ax-3)=\lim_{x\to -2+}(x^2-2x)$$
$$a(-2)-3=(-2)^2-2(-2)$$
$$-2a-3=4+4$$
$$-2a=11$$
$$a=-\frac{11}{2}$$

A. $a=\frac{5}{2}$ B. $a=-\frac{11}{2}$

C. $a=-\frac{5}{2}$ D. $a=-\frac{3}{2}$

E. No value of a will make the function have a point discontinuity at $x = -2$.

Use the graphs of $f(x)$ and $g(x)$ pictured below to answer question 8 and 9.



8. Which of the following statements is/are true about the graphs of $f(x)$ and $g(x)$?

I. $f(x)$ is increasing on the intervals $(-5, -2)$ and $(2, 7)$.

II. $f(x) = g(x)$ only at $x = 5$.

III. $f(x) > g(x)$ only on the interval $(5, 6)$.

A. I only
B. I and II only
C. I, II and III
D. II and III only
E. II only

9. If $p(x) = 2mx^2 - 3x$, for what value(s) of m would $p(-1) = f(g(0))$?

$$2m(-1)^2-3(-1)=f(g(0))$$
$$2m(1)+3=f(2)$$
$$2m+3=0$$
$$2m=-3$$
$$m=-3/2$$

A. $m=-\frac{1}{2}$ B. $m=\frac{5}{2}$

C. $m=-\frac{3}{2}$ D. $m=\frac{3}{2}$

E. No value of m would make $p(-1) = g(f(0))$.